Modelling of high-dimension dynamics by random dynamical systems
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The majority of natural systems are known to be open, i.e., subject to numerous external forcings; these forcings can be modeled by random dynamical systems (RDS). The RDS present a necessary and important step towards reconstructing the observed systems when their adequate first-principle mathematical models are either unknown or subjected to further verification. Note that, even for deterministic systems, the construction of a deterministic model from the observed TS and use of this model for prediction has quite a number of principal restrictions. First, according to the Takens' theorem, the reconstruction of a phase trajectory is possible in a phase space of sufficiently high dimension dE > 2dS+1, where dS is the phase space dimension of the system that has generated the initial TS. This means that a deterministic dynamical system (DDS) describes correctly behavior of the reconstructed system in the subspace of dimension dS that is much smaller than the dimension of the phase space dE of the model. Consequently, the model is not adequate for the system at relatively small changes of control parameters. The second restriction is the limitation on prior information. To confirm determinism of the observed system one has to ensure that the attractor reconstructed by the TS has a finite dimension and to find the smallest embedding dimension for this attractor. The available methods of determining such dimensions are inapplicable for analysis of the TS generated by real systems. Reconstruction in the form of RDS (stochastic model) removes these restrictions, thus making the proposed approach more universal. A basic idea underlying the stochastic approach is that the robust dynamic properties of the system evolution can be described by a few equations, while other features may be considered as a stochastic disturbance. A principal new step here is inclusion of parameterized stochastic perturbation in the model of the evolution operator; it allows us to significantly expand a class of reconstructed systems. The method of parameterization of such models on the basis of artificial neural networks is developed, as well as technique of investigation of model parameter space is suggested. Possibilities of the approach with reference to the analysis of time series generated by high dimensional dynamic systems are demonstrated by model examples. In particular, the prediction of changes of characteristics of observed process is constructed. Possible other applications of the method are discussed.

Key Words: random dynamical systems, prognosis of qualitative behaviour