On the dynamics of classical interacting spins
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We consider the general classical Heisenberg model (HM) with z-axis anisotropy and external magnetic field and show that its phase space is foliated into a family of invariant manifolds diffeomorphic to \((S^2)^\Lambda\). The flow on each leaf \(S\) is Hamiltonian. Subsequently, we focus on the isotropic HM in the absence of external field. We discuss the rotational symmetry of the model and further analyze its phase space structure. We prove that the manifold \(F\) of longitudinal fixed points (LFPs) intersects each leaf \(S\) orthogonally. For a real autonomous dynamical system with a continuous symmetry, we show that, under some general conditions, Lyapunov stability of fixed points (FPs) and orbits on an invariant subspace is extended to the whole phase space. We apply this theorem to the case of the isotropic HM to prove that the ferromagnetic (FR) state and the antiferromagnetic (AF) state with non-zero total spin are both stable FPs. The theorem breaks down at total spin zero, and as an example, the AF state on an equal-spins leaf is found to be unstable.

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