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Chaos in an infinite-dimensional dynamical system

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Abstract

Let H be a Hilbert space, $A = A^* \geq m > 0$ be a selfadjoint operator in H, possibly unbounded. Consider the following dynamical system:

$$\dot{u} = -Au + bu, \quad b := b(t) := b(u(t)) = \frac{(Au, u)}{(u, u)}, \quad u(0) = u_0; \quad \dot{u} = \frac{du}{dt}. \tag{1}$$

Here (u, v) is the inner product in H, $u_0 \in D(A)$, $D(A)$ is the domain of A. Let us formulate our basic results:

1. Problem (1) has a unique solution defined for all $t > 0$ and $||u(t)|| = ||u_0|| \ \forall t > 0$.

2. The solution can be found analytically:

$$u(t) = \frac{e^{-tA}u_0}{(1 - 2\int_0^t h(s)ds)^{1/2}}, \quad h(t) := (Ae^{-2tA}u_0, u_0). \tag{2}$$

Without loss of generality one may assume that $||u_0|| = 1$, and then the solution can be rewritten as:

$$u(t) = \frac{\int_m^\infty e^{-ts} dE_s u_0}{(\int_m^\infty e^{-2ts} d\rho(s))^{1/2}}, \quad \rho(s) := (E_s u_0, u_0), \tag{3}$$

where $E_s = E((-\infty, s])$ is the resolution of the identity of A.

3. If the interval $[m, m+\delta]$ does not contain eigenvalues of A and is filled with an absolutely continuous spectrum of of A, and if $E([m, m+\delta])u_0 \neq 0$, then the solution to (1) *does not have a strong limit in* H *and does not remain in any fixed finite-dimensional subspace of* H.

In this sense the solution exibits chaotic behavior.

4. If m is an isolated eigenvalue of A, P_m is the orthoprojector on the corresponding eigenspace H_m of A, and if $P_m u_0 \neq 0$, then the limit $\lim_{t\to\infty} u(t) = \phi$ exists, $\phi \in H_m$, and $||\phi|| = ||u_0||$.