Nonlinear Shallow Water Equation in Polar Coordinates

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An interaction of two water waves in a circular basin is studied within quadratic approximation. When the polar coordinates are used, the usual perturbation techniques in separation of variables method inevitably lead to a series of overdetermined systems of linear algebraic equations for unknown coefficients (in contrast with the Cartesian coordinates). However, if we formally introduce a new function satisfying the first system of this series, all these overdetermined systems become compatible (remaining overdetermined) for the special case of the nonlinear shallow water equation. Using the new function and quadratic polynomials of the Bessel functions of radius, we explicitly express the coefficients of the resulting harmonics. It gives solutions describing the two-waves interaction which are found with the same accuracy as the nonlinear shallow water equation is derived. As a consequence, a general boundary problem Boundary problem can be explicitly solved in these terms.

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