We investigate the behaviour of chaotically forced nonlinear mapping and show that under specific conditions it is possible to obtain stable aperiodic dynamics. Subsequent to a blowout bifurcation, the response dynamics is nonchaotic and is confined to a limit set which is nonsmooth or strange. There is dependence of the dynamics on the initial conditions of the chaotic drive. The attracting limit set can be characterized through parameter sensitivity exponents, which in turn are related to the distributions of finite-time Lyapunov exponents. Generation of nonchaotic aperiodic dynamics is of potential importance in applications such as secure communications, and in understanding the dynamics of biological systems.