Modeling and Simulation of Self-Organized Criticality in Landslides

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Abstract: The paper elaborates on an avalanche-like dynamic model for catastrophic landslides introducing the effect of water diffusion along the failure plane. The main idea lies on the assumption that the stochastic nature of water diffusion along the failure plane results in a dynamic decrease in time of the shear strength for the entire rock mass parallel to this plane. To this end, a single stochastic constitutive equation is proposed, modeling external and internal stresses, spatial interactions between neighborhood sites as well as water diffusion, which are shown to reproduce correctly experimental observations. Indeed, simulations of a discrete automaton were performed, in order to study the model dynamics. It is demonstrated that the model exhibits features of self-organized critical behavior and solves the reported discrepancy between simulations outcomes and experimental data for the corresponding power law exponent.

Keywords: Catastrophic landslides, Water diffusion, Discrete automaton, Self-Organized Criticality.

1. Introduction

Landslide is the movement of a mass of rock or debris down a slope, which has one or more distinct failure surfaces. In this paper catastrophic landslides on a discrete failure plane are considered. Their velocities are typically on the order of 1m/day up to 300 Km/hr (when they are generated by earthquakes). Landslide activity increases after prolonged rains, or after the frost leaves the ground in the spring. Landscape evolution arises from the integrated effect of erosion and mass transfer over geological spatial and temporal scales, as well as from the effect of water presence into the rock mass. Concerning the ground water, the most important effect in a rock mass is the reduction in stability resulting from water pressures within the discontinuities. The landslides are related with the perturbation of mass balance of rock mass, and they are caused by natural reason (i.e. earthquake, intense rainfall), gradual attenuation of mechanic properties of the rock mass, or even human intervention (i.e. excavations). The dimensions of landslides are exceptionally various. In general, they are large-scale movements with
appreciable quantities of geomaterials, capable to cause wide destructions. There are two groups of landslides: a) translational, where they slide along a bedding plane or other plane of weakness in the rock. In this case the entire mass moves parallel to that plane, and b) rotational landsides which develop in surficial deposits or weathered rock. [D.C. Wyllie, and C.W. Mah, 2008].

The main failure mechanism of the landslides is that shear takes place along either a discrete sliding surface, or within a zone, behind the face. If the shear force (driving force) is greater than the shear strength of the rock (resisting force) on this surface, then the slope will be unstable. Instability could take the form of displacement that may or may not be tolerable, or the slope may collapse either suddenly or progressively. The stability of slope can be expressed via the following terms: a) factor of safety (FS), stability quantified by limit equilibrium of the slope, which is stable if FS>1, b) strain, failure defined by onset of strains greater enough to prevent safe operation of the slope, c) probability of failure, stability quantified by probability distribution of difference between resisting and displacing forces, which are each expressed as probability distributions, and d) load and resistance factor design, stability defined by the factored resistance being greater than or equal to the sum of the factored loads [P.H. Rahn, 1996].

In this article we focus in modeling and simulation of landslide phenomenon by means of the notion of self-organized criticality (SOC). The idea is not new and many remarkable attempts has been done in the past (see for example [Turcotte et. al., 2002] and references therein) since the original work by Per Bak [P. Bak, et. al., 1987]. More specifically the problem of the correct power law exponent is addressed. Indeed, while most of the SOC models proposed in the past correctly reproduce the power law behavior of landslides evolution there is a discrepancy between the exponent of the noncumulative distribution function from simulation outcomes which is around 1 and the exponent obtained from landslides experimental data which
is around 2 [S. Hergarten and H. J. Neugebauer, 2000]. In the next section we elaborate on the recent work [M. Zaiser, et. al., 2004] on slope failure by introducing the water spatial diffusion in the failure plain as an appropriate internal variable which correctly shifts the exponent in higher values in a 2-D simulation code.

2. Modeling Water Diffusion

We consider the problem of landslide as it is depicted in Fig. 1. In order to describe the evolution of the landslide in time and space the following simulation code was build. In a 2-D cellular automaton of regular grid the dynamic variables of the shear stress $\tau_{ij}$ and shear strain $u_{ij}$ are assigned to each node $i, j$ of the lattice. Starting from initial condition of zero external stress the stress of each site grows linearly with time by a small amount. Increase of local stress results in a corresponding increase of the local strain, and as a result the site becomes excited, if the following constitutive inequality holds (for a more rigorous introduction of the model see [M. Avlonitis, and G.Efremidis, in preparation]),

$$\tau_{ij} - \tau_{ij}^s > 0 ,$$

(1)

where $\tau_{ij}^s$ is the local shear strength of the slope. Moreover $\tau_{ij}$ is considered as the sum of the externally increasing applied stress $\tau^{ext}$ and an internal stress $\tau^{int}_{i,j}$ which is the result of stress redistribution because of local spatial interactions. The condition for a site to be excited now reads as follows,

$$\tau^{ext} + \tau^{int}_{i,j} - \tau_{ij}^s > 0 .$$

(2)

If a second order gradient expression of the internal stress is adopted (see [M. Zaiser, et. al., 2004]),

$$\tau^{int}_{i,j} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} ,$$

(3)

the final condition for a site to be excited is,

$$\tau^{ext} + (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}) - \tau_{ij}^s > 0 .$$

(4)

For each external stress increase, if an arbitrary site becomes excited its strain is increased by a discrete amount. After all sites are updated, the new stresses are computed, and the procedure is repeated until no site is excited for the same external stress. The number of affected sites defines the size $s$ of the produced avalanche. The automaton continues with a new increase of the external stress until all sites become excited in the same avalanche i.e. until the slope failure.
In order to mimic water diffusion along the failure plane, in this article we relax the condition for constant local shear strength in time. Indeed, after the strain update procedure in automaton sites, an evolution of water diffusion is taking place by decreasing the local shear strength by a constant amount in an arbitrary site in space. In the course of time, the vicinity of the initial site with non-zero water density is updating with lowering by the same amount of the shear strength. With this way we mimic water diffusion in the failure plane as a result of random soil cracks in the slope face.

3. Simulation Results and Discussion

Fig. 2 shows the result of an automaton simulation in the case of constant shear strength in time but not in space, allowing for spatial variation because of non-uniformities on the failure plain. It is seen that the power exponent of the non-cumulative distribution of avalanche sizes is close to 1, as founded in literature in previous works, e.g.,

$$P(s) \propto s^{-\alpha}, \quad \alpha \approx 1.$$  (5)

The distribution for different system size is depicted as well (grids of 25x25 and 35x35 are depicted), confirming that the simulating system manifests self organized criticality. Indeed, the distribution follows a power law with an exponent around 1, while above a critical avalanche size decays rapidly, the size of the critical avalanche is being limited by the system size.

Figure 2. Non-cumulative distribution of avalanche sizes on grids of different sizes.
Fig. 3 shows the result of an automaton simulation where water diffusion in the failure plane is present. The distribution follows, as before, a power law indicating self-organized critical behavior, but deviating from the non-water diffusion case in two points. First, small deviations from an ideal power law shape over the range of avalanche’s size are observed as well as deviation at the beginning of the process is observed. These deviations are in accordance with real experimental data for landslides (see for example [Y. Fuyii, 1969], [Turcotte et. al., 2002]). The deviations can be explained as a consequence of non-uniformities in the failure plane, the effect of which is manifested because of the presence of water diffusion. Second, the exponent of the power law is shifted to higher values, here around 2, again in agreement with real data from landslides measurements.

As an epilogue of this preliminary work it is noted that a more complete presentation of the arguments introduced in this article can be found in a forthcoming paper by the authors. We believe that the introduction of water diffusion as an extra internal variable not only correctly reproduces landslide phenomena, but also can be applied to other open problems exhibiting self-organized criticality as well, where discrepancy to power exponent is also reported, e.g. earthquakes.
References


