Linear Communication Channel Based on Chaos Synchronization

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Abstract: This paper presents the possibility of designing a linear communication channel by modulating chaotic analog systems. After presenting the general setup, conditions for correct demodulation and linear dynamic input-output behavior are demonstrated. For a linear dynamic relation between the modulating and demodulated signals, channel equalization is used to achieve wider bandwidth transmission. The presented case studies, regarding the Lorenz and Chen systems, highlight the applicability of the proposed method for high speed digital communication. The overall performance of the resulting communication system is analyzed in terms of speed, security and occupied frequency bandwidth. The concluding remarks point towards some directions in further research.

Keywords: Chaos synchronization, Channel equalization, Lorenz system, Chen System.

1. Introduction

The present contribution aims at characterizing an analog modulation method to build a wide-band communication channel. The proposed approach is based on chaos synchronization between the emitter and receiver ends in order to achieve a supplementary level of security under the standard digital encryption layer. The general synchronization setup is based on the system partitioning method, first introduced by Carroll and Pecora in [1]. Considering some reasonable restrictions for the chaotic system, we develop a method for the characterization of the input-output relation of the proposed communication channel, namely between the modulating signal applied at the emitter end and the demodulated signal obtained at the output of the receiver. The linear dynamic system that models the studied relation shows the frequency limitations for the modulating signal. In order to increase the transmitted signal bandwidth, a feed forward equalizer is proposed and its efficiency studied. Case studies regarding third order chaotic systems of the Lorenz [2] and Chen [3] types aim at proving the feasibility of the proposed method. The Lorenz system, chosen for implementation advantages [4] and for synchronization simplicity [5-6], proved to be a good prototype for the proposed method, allowing easy synchronization and demodulation. In order to apply the general setup to the Chen system, some coefficient modifications had to be made to allow the proposed demodulation scheme.

The next section concentrates on the demonstration of the proposed method in the general case. The error dynamics method is used, leading to the conclusion that the input – output relation of the proposed channel is linear. Simulation results, confirming the theoretical solution, are presented in the third section, for two case studies.

2. General Results

The communication channel based on chaos synchronization is built around a nonlinear chaotic emitter described by the general state equations:

$$\mathbf{x}' = \mathbf{f}(\mathbf{x})$$

where $\mathbf{x}$ denotes the N-dimensional state vector and $\mathbf{f}:\mathbb{R}^N \to \mathbb{R}^N$ is the nonlinear state transition function.

In order to achieve chaos synchronization, using the emitter subsystem approach, we presume that the above state equations can be partitioned in the form:

$$\begin{cases} x_1' = f_{11}(x_1) + f_{12}(\mathbf{x}_R) \\ \mathbf{x}_R' = \mathbf{f}_{21}(x_1) + \mathbf{f}_{22}(\mathbf{x}_R) \\ y = x_1 \end{cases}; \ \mathbf{x} = \begin{bmatrix} x_1 \\ \mathbf{x}_R \end{bmatrix}; \quad \begin{matrix} f_{11}:\mathbb{R}\to\mathbb{R}; f_{12}:\mathbb{R}^{N-1}\to\mathbb{R}; \\ \mathbf{f}_{21}:\mathbb{R}\to\mathbb{R}^{N-1}; \mathbf{f}_{22}:\mathbb{R}^{N-1}\to\mathbb{R}^{N-1} \end{matrix}$$

If we choose to transmit the first state variable, x_1, the synchronizing receiver results in the form:

$$\tilde{\mathbf{x}}_R' = \tilde{\mathbf{f}}_{22}(\tilde{\mathbf{x}}_R) + \tilde{\mathbf{f}}_{21}(y)$$

where the $\tilde{}$ signals and functions denote the receiver correspondents of the emitter ones.

Presuming the receiver has been correctly designed, in the case of parameter matching ($\tilde{\mathbf{f}}_{21}(\cdot) = \mathbf{f}_{21}(\cdot); \tilde{\mathbf{f}}_{22}(\cdot) = \mathbf{f}_{22}(\cdot)$), the error dynamics is globally asymptotically convergent to zero:

$$\boldsymbol{\varepsilon}' = \tilde{\mathbf{f}}_{22}(\tilde{\mathbf{x}}_R) - \mathbf{f}_{22}(\mathbf{x}_R); \quad \boldsymbol{\varepsilon}(t) \to 0$$

where the error $\boldsymbol{\varepsilon}(t) = \mathbf{x}_R(t) - \tilde{\mathbf{x}}_R(t)$ denotes the difference between the emitter and receiver state vectors.

To achieve transmission of the useful information signal, m(t), a direct modulation approach is used at the emitter end, by modifying the first state equation:

$$x_1' = f_{11}(x_1) + f_{12}(\mathbf{x}_R) + m(t)$$

At the receiver end, demodulation is implemented by appending the previous receiver state equations with a similar one, without modulation:

$$\tilde{x}_1' = \tilde{f}_{11}(\tilde{x}_1) + \tilde{f}_{12}(\tilde{\mathbf{x}}_R)$$

The demodulated signal is algebraically obtained giving the receiver output equation:

$$\tilde{m}(t) = y(t) - \tilde{x}_1(t)$$

The dynamic time evolution of the demodulated signal is governed by the resulting nonlinear differential equation, obtained by subtracting the previous state equations:

$$\tilde{m}'(t) = f_{11}(x_1) - \tilde{f}_{11}(\tilde{x}_1) + f_{12}(\mathbf{x}_R) - \tilde{f}_{12}(\tilde{\mathbf{x}}_R) + m(t)$$

As we presumed the error dynamics to be globally asymptotically stable, in the case of parameter matching ($\tilde{f}_{11}(\cdot) = f_{11}(\cdot); \tilde{f}_{12}(\cdot) = f_{12}(\cdot)$), the second difference converges to zero

$$f_{12}(\mathbf{x}_R) - \tilde{f}_{12}(\tilde{\mathbf{x}}_R) \to 0$$

After the synchronizing transient, the demodulated signal differential equation is simplified to the form:

$$\tilde{m}'(t) = f_{11}(x_1) - \tilde{f}_{11}(\tilde{x}_1) + m(t)$$

Assuming the average value of the modulating signal is zero, we can decompose the algebraic nonlinear function, $f_{11}(.)$ in a power series around this value, resulting:

$$f_{11}(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n f_{11}}{dx^n}(0) \cdot x^n$$

If the nonlinear function, $f_{11}(.)$, is smooth enough and the modulating signal small, we may neglect the higher order terms, to obtain:

$$\tilde{m}'(t) \simeq \frac{df_{11}}{dx}(0) \cdot \tilde{m}(t) + m(t); \quad \tilde{m}(t) = x_1(t) - \tilde{x}_1(t)$$

The resulting linear dynamic system, characterizing the proposed communication channel based on chaos modulation, is stable if the structural constant, *a*, is negative:

$$a = \frac{df_{11}}{dx}(0) < 0$$

In the complex frequency domain, the input-output characterization of proposed communication channel is given by the transfer function:

$$H(s) = \frac{\tilde{M}(s)}{M(s)} \simeq \frac{1}{s+a}$$

The obtained result shows that the modulating signal cannot be perfectly recovered unless the maximum frequency in its spectrum is (much) less than the structural constant, *a*. Aiming our results at applications in higher frequency transmission, we propose a single order equalizer connected in a feed forward topology. The real zero of the equalizer must compensate the channel pole, while the pole of the equalizer is chosen at a much larger frequency, ω_C, to limit the overall pass band of the communication system.

$$H_E(s) = \frac{s+a}{1/\omega_C \cdot s + 1}$$

The overall communication system is linear, characterized by the transfer function:

$$H_E(s) = \frac{\omega_C}{s + \omega_C}$$

3. Case Studies

3.1. *The Lorenz system*

In this particular case, the emitter subsystem is a third order, analogue, Lorenz type system:

$$\begin{cases} x' = \sigma \cdot (y - x) \\ y' = \rho \cdot x - y - x \cdot z \\ z' = x - \beta \cdot z + x \cdot y \end{cases}$$

For a large enough parameter range, this system exhibits chaotic behavior, useful in ensuring the secrecy the modulating signal. To achieve synchronization, the y state variable is transmitted to the receiver subsystem, designed by using the system partitioning method:

$$\begin{cases} \tilde{x}' = \sigma \cdot (y - \tilde{x}) \\ \tilde{z}' = \tilde{x} - \beta \cdot \tilde{z} + \tilde{x} \cdot y \end{cases}$$

The second equation, for $\tilde{y}'$, will be introduced later on in the state description of the receiver for further use in the modulation/demodulation process.

Taking the autonomous case, $y = 0$, its state equations become linear, with a state transition matrix of the form:

$$A = \begin{pmatrix} -\sigma & 0 \\ 1 & -\beta \end{pmatrix}$$

The resulting eigenvalues are real and negative, for positive σ and β, ensuring global asymptotic stability of the receiver. The error dynamics can be analyzed by subtracting the receiver equations from their emitter counterparts. The state errors, $\varepsilon_1 = x - \tilde{x}$, $\varepsilon_3 = z - \tilde{z}$, are described by the dynamic equations:

$$\begin{cases} \varepsilon_1' = -\sigma \cdot \varepsilon_1 \\ \varepsilon_3' = \varepsilon_1 - \beta \cdot \varepsilon_3 + \varepsilon_1 \cdot y \end{cases}$$

Due to the fact that all state variables are bounded, it is easy to demonstrate iteratively that all errors decay exponentially to zero. Thus the proposed receiver synchronizes with the chaotic emitter. In order to use this synchronizing setup to transmit an information signal, $m(t)$, by the direct modulation technique, we add the modulating signal to the second equation of the emitter:

$$\begin{cases} x' = \sigma \cdot (y - x) \\ y' = \rho \cdot x - y - x \cdot z + m(t) \\ z' = x - \beta \cdot z + x \cdot y \end{cases}$$

At the receiving end of the communication channel, we demodulate the received signal, $y(t)$, by subtracting from it the locally recovered second state variable, $\tilde{y}$:

$$\tilde{y}' = \rho \cdot \tilde{x} - \tilde{y} - \tilde{x} \cdot \tilde{z}$$

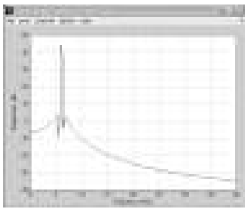

Figure 1.a The PSD of the modulating signal

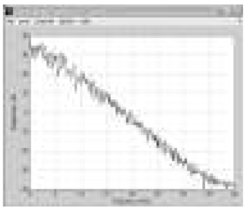

Figure 1.b The PSD of the chaotic signal for amplitude 10

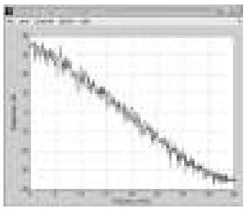

Figure 1.c The PSD of the chaotic signal for amplitude 20

The error dynamics for the second state variable:

$$\varepsilon_2' = \rho \cdot \varepsilon_1 - \varepsilon_2 - \left(x \cdot z - \tilde{x} \cdot \tilde{z}\right) + m$$

leads to the differential equation for the demodulated signal:

$$\tilde{m}' = -\tilde{m} - \left(z - \rho\right) \cdot \varepsilon_1 - \tilde{x} \cdot \varepsilon_3 + m$$

After the synchronization transient died out, the demodulator equation can be approximated as follows:

$$\tilde{m}' \simeq -\tilde{m} + m$$

This highlights the linear memory character of the resulting communication channel, which can be characterized by the transfer function:

$$H(s) = \frac{\tilde{M}(s)}{M(s)} \simeq \frac{1}{s+1}$$

The dynamic range of the amplitude for the modulating signal must ensure both linearity of the communication system and the lack of visibility of the modulating signal in the transmitted chaotic one. In order to track the visibility of the modulating signal in the transmitted chaotic one, we studied the power spectral density (PSD) of the transmitted signal, for different amplitudes of the modulating signal, depicted in figure 1.

As seen from the simulation results, if the amplitude of the modulating signal is as high as 10, its PSD is not visible in the one of the transmitted signal. Beginning with amplitudes of the order of magnitude 20, the modulator line becomes visible in the PSD of the chaotic signal. This gives a dynamic range of modulating signal amplitude comparable to the amplitudes of the state variables and transmitted signal, thus large enough for practical applications.

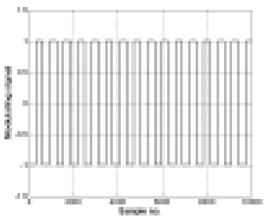

Figure 2.a The modulating signal

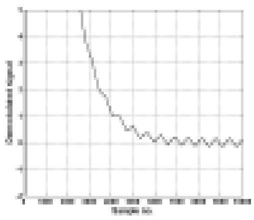

Figure 2.b The demodulated signal without equalizer

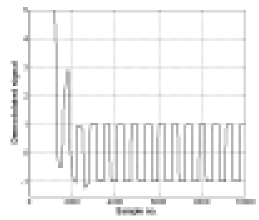

Figure 2.c The demodulated signal with equalizer

To increase the bandwidth of the transmitted signal, we use for our equalizer a transfer function of the form:

$$H_E(s) = \frac{s+1}{1/\omega_C \cdot s+1} = \frac{s+1}{0.01 \cdot s+1}$$

In figure 2, we give example results showing the demodulated signal without and with the feed forward equalizer.

3.2. *The Modified Chen system*

The Chen system is described by the state equations:

$$\begin{cases} x' = -a \cdot x + a \cdot y \\ y' = (c-a) \cdot x + c \cdot y - x \cdot z \\ z' = -b \cdot z + x \cdot y \end{cases}$$

where the standard values for the coefficients are $a = 35$, $b = 3$ and $c = 28$.

To achieve synchronization, the second state variable must be transmitted and the synchronizing receiver is built around the first and third state equations. Due to the fact that the y coefficient in the second state equation, c, is positive, the proposed demodulation method cannot function since it will lead to demodulator instability. In order to use our approach on the Chen system, some coefficient modifications need to be made:

$$\begin{cases} x' = -a \cdot x + a \cdot y \\ y' = d \cdot x - c \cdot y - x \cdot z \\ z' = -b \cdot z + x \cdot y \end{cases}$$

As a consequence of the sign change of the c coefficient, the values for the other ones have to be modified to achieve chaotic behavior. For instance, $a = 15$, $b = 3$, $c = 1$ and $d = 32$ is a set of values leading to a chaotic attractor, as seen from the simulation results given in figure 3, where the 3D system trajectory suggests chaos.

The resulting receiver is described by:

$$\begin{cases} \tilde{x}' = -\tilde{a} \cdot \tilde{x} + \tilde{a} \cdot y \\ \tilde{z}' = -\tilde{b} \cdot \tilde{z} + \tilde{x} \cdot y \end{cases}$$

In the autonomous case, the receiver state equations are linear and the state transition matrix is:

$$A = \begin{pmatrix} -a & 0 \\ 0 & -b \end{pmatrix}$$

Obviously, the synchronizing receive is globally asymptotically stable, with negative eigenvalues $-a$ and $-b$.

The error dynamics, in the case of parameter match, is given by:

$$\begin{cases} \varepsilon_1' = -a \cdot \varepsilon_1 \\ \varepsilon_3' = -b \cdot \varepsilon_3 + \varepsilon_1 \cdot y \end{cases}$$

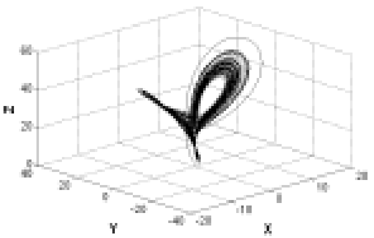

Figure 3 The modified Chen attractor

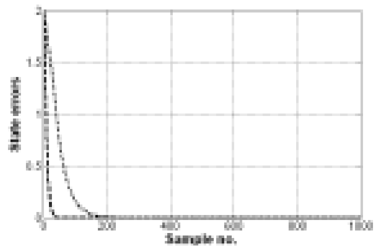

Figure 4 Time evolution of the state synchronization errors

The state errors, $\varepsilon_1 = x - \tilde{x}$, $\varepsilon_3 = z - \tilde{z}$, exponentially converge to zero, with the time constants –1 / a and –1 / b.

The demodulator is based on the second state variable:

$$\tilde{y}' = \tilde{d} \cdot \tilde{x} - \tilde{c} \cdot \tilde{y} - \tilde{x} \cdot \tilde{z}$$

Subtracting from second equation of the emitter, the demodulated signal dynamics results in the form:

$$\tilde{m}' = -c \cdot \tilde{m} - \left(z - d\right) \cdot \varepsilon_1 - \tilde{x} \cdot \varepsilon_3 + m$$

After the synchronization transient, of time length τ_{Tr} = Max(–1 / a, –1 / b), the state errors become negligible, and the time evolution of the recovered signal can be approximated by:

$$\tilde{m}' \simeq -c \cdot \tilde{m} + m$$

The corresponding transfer function results in the form:

$$H(s) = \frac{\tilde{M}(s)}{M(s)} \simeq \frac{1}{s + c}$$

Following the general method, the feed forward equalizer has the transfer function:

$$H_E(s) = \frac{s + c}{1/\omega_C \cdot s + 1} = \frac{s + 1}{0.01 \cdot s + 1}$$

Simulation results confirm the design calculations previously presented. For the same parameter values as above, the synchronization errors decay exponentially with the predicted time constants, as presented in figure 4.

The efficiency of the proposed equalizer is highlighted in figure 5, where the modulating signal and the demodulated one are compared.

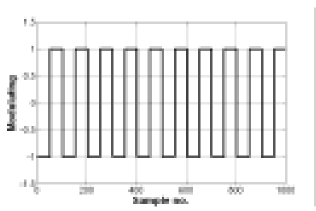

Figure 5.a. The Modulating signal

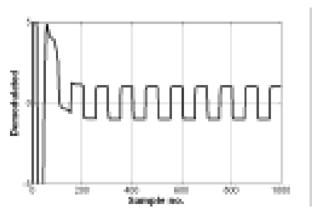

Figure 5.b. The demodulated signal, using the equalizer

5. Conclusions

We proposed an analog wide-band communication channel based on the chaos synchronization and direct modulation principles. Using the system partitioning synchronization and error dynamics methods, we demonstrated, in a general enough setup that a linear dynamic relation exists between the modulating signal and its demodulated counterpart. A feed-forward channel equalization technique ensures that the modulating signals can have large enough bandwidth, leading to the possibility of high speed digital communication. The presented case studies highlight the feasibility of the general method.

Further research is needed to approach noise and parameter mismatch problems, specific to analog implementation of synchronizing transmissions. A possible approach can be to extend the use of the communication setup to digital modulation.

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