Color Reconstruction and Image Segmentation by Logic Theory via Variant Operator

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Abstract: The proposed model introduces an improvement over some other basic color classification techniques which are considered more challenging to color segmentation methods. In color vision systems require a first step of classifying pixels in a given image into a discrete set of color classes. In this paper we describe a human perception based approach to pixel color segmentation which applied in color reconstruction by numerical method associated with graph-theoretic image processing algorithm typically in grayscale.

Fuzzy sets defined on the Hue, Saturation and Value components of the HSV color space, provide a fuzzy logic model that aims to follow the human intuition of color classification.

Keywords: Color reconstruction, Image segmentation, Fuzzy logic, HSV color space.

1. Introduction

Many color vision systems require a first step of classifying pixels in a given image into a discrete set of color classes. This early vision step plays an important role in computer vision applications, as error in this process will be propagated further [13]. However, while humans can easily classify colors in the spectrum visible to the human eye, machines find this task more challenging. Although many color segmentation methods have been proposed [5-7, 9, 17-19], no algorithm has been proven to provide an optimal solution, and research efforts are being continued.

Numerical methods associated with graph-theoretic image processing algorithms often reduce to the solution of a large linear system. We show here that choosing a topology that yields a small graph diameter can greatly speed up the numerical solution.

2. Fuzzy logic model and Graph theory

Human perceived color defines a three-dimensional color space, called HSV, which corresponds to two cylindrical cones joined at their base, as shown in Figure 1 (left). Our brains use the following notions: Hue (the actual color, e.g., “blue”, “yellow”, “orange” etc., as defined by a radial value around a color wheel); Saturation (the vividness or dullness of the color); Value (the lightness or darkness of the color). Color naming, or color categorization, has been based on segmentation of the HSV color space using a fuzzy logic model that follows a human intuition of color classification.
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While some common approaches are based on sampling HSV triples using fixed-size bins [1], the method described in this paper predefines the segments and color reconstruction using a fuzzy logic model, and divides the color space into segments based on linguistic terms by the data on Lab color space, Figure 1 (right). This approach is different than some data-driven approaches such as nearest neighbour classification, in which the shapes of the segments are determined by the distribution of the samples in the training set, or basic approaches such as color space thresholding, that defines the segment shapes based on the data structures used by the algorithm [10].

![Figure 1: The HSV color spaces](image)

Fuzzy Logic is often used as an interface between logic and human perception [2, 4, 7, 12, 20, 21]. The method is based on fuzzy logic modeling of the HSV color space, which is more intuitive and closer to the human perception of color than the RGB space [14]. The reasoning procedure is based on a zero-order Takagi-Sugeno model [16]. A fuzzy set is described by its membership function. In color naming, we can consider that any color category, \( C_k \), is a fuzzy set with a membership function, \( f_{C_k} \), which assigns to each color sample \( x \) a membership value \( f_{C_k}(x) \) within the \([0, 1]\) interval. This value represents the certainty we have about \( x \) has to be named with the linguistic term, \( t_k \), corresponding to category \( C_k \). From this point of view, the first step of any color-naming modeling process will be the definition of the membership functions for each color category. Once these functions are defined, it will be possible to compute a color descriptor, \( CD(x) \), such as the following: \( CD(x) = (f_{C_1}(x), \ldots, f_{C_n}(x)) = (m_1, \ldots, m_n) \), where \( m_k \in [0, 1] \) for \( k = 1, \ldots, n \) and \( \sum_k m_k = 1 \). \( CD(x) \) describes the membership relation of \( x \) to...
each color category, $m_k$ is the certainty value associated to $x$ by $f_k$ and $n$ is the number of categories considered. To test the stability of the method we name different color regions in the HSV color space via the transition of color names along the black-red and purple-white lines along the “color circle” in the HSV space in [18, 19]. Our model considers the color-naming task as a fuzzy decision. In our case $n = 11$ and the categories considered in the model correspond to the 11 basic color terms proposed by Berlin and Kay [2, 4], that is $\{\text{white, black, red, green, yellow, blue, brown, purple, pink, orange, gray}\}$. The information contained by $\text{CD}(x)$ can be used by a decision function $N(x)$ that assigns to $x$ one of the 11 color terms considered. At the moment, the decision function we have chosen assigns the color term that corresponds to the category $C_k$ with the highest membership value $m_k$ in $\text{CD}(x)$ as follows: $N(x) = t_k m_k = \max_{C_k} \{f_k(x)\}$. Therefore, the color descriptor $\text{CD}(x)$ defined above is a vector of 11 components and the information contained in such descriptor can be used by a decision function to decide the color name of a given stimulus $x$. See our algorithm to calculate the above variant in [13, 14].

A graph is a pair $G = (V; E)$ with vertices $v \in V$ and edges $e \in E \subseteq V \times V$. An edge, $e$, spanning two vertices, $v_i$ and $v_j$, is denoted by $e_{ij}$ [12, 15]. Let $n = |V|$ and $m = |E|$ where $\mid$ denotes cardinality. A weighted graph has a value (typically nonnegative and real) assigned to each edge called a weight. The weight of edge $e_{ij}$ is denoted by $w(e_{ij})$ or $w_{ij}$. Conjugate gradients is generally the algorithm of choice for solving a large, sparse, system of linear equations, so when applied to a matrix generated as a result of graph topology (e.g., Laplacian matrix, adjacency matrix), such that the rate of convergence for the conjugate gradients method is a function of the graph diameter. The diameter of a graph, $G$, is defined formally as:

\[
\text{Diameter} (G) = \text{MAX} \left\{ \min_{v_i, v_j \in F} (n(v_i, v_j)) \right\}, \text{ where } n(v_i, v_j) \text{ denotes the number of nodes traversed in the shortest path between two nodes (i.e., the length of the minimal geodesic between nodes } v_i \text{ and } v_j \right\} [5,12, 15]. \text{ In other words, the graph diameter is the maximum number of nodes traversed along an optimal path connecting two arbitrary nodes. After } e_{ij} \text{ has been arbitrarily assigned an orientation, define the } m \times n \text{ edge-node incidence matrix } A \text{ and the } m \times m \text{ constitutive matrix } C \text{ as the diagonal matrix with the weight of each edge along the diagonal as well as the } n \times n \text{ Laplace-Beltrami operator } L \text{ with relation } L = A^TCA, \text{ where } A^T \text{ is transpose matrix of } A \text{ with the following components:}
\]
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\[
A_{ej} = \begin{cases} 
1 & \text{if } i = k \\
-1 & \text{if } j = k \\
0 & \text{otherwise}
\end{cases}
\]

\[
L_{vi} = \begin{cases} 
0 & \text{if } e \in E.
\end{cases}
\]

There are four distinct reasons to employ graph theoretic approaches to image segmentation [5]: (i). local-global interactions, (ii). new algorithms, (iii). adaptive sampling, (iv) new architectures for image processing. Therefore in a computational context, the above reasons suggest the use of graph-theoretic data structures, rather than pixels, for example see Figures 2 and 3. In turn, the flexible data structures based on graphs, which are familiar in computer graphics, have been relatively unexplored in computer vision, so the structure and algorithms of graph theory provide a natural language for space-time adaptive sensors.

![Figure 2](image1.png)  ![Figure 3](image2.png)

**Figure 2** A fingerprint image (left) and its orientation field represented by a graph (right) [5]

**Figure 3**: Topology of the connected pyramid graph with 4-connected (a), 8-connected (b), and radius = 5 connected (c) within-level connections [5].

3. **Image Segmentation and Reconstruction in the Gray scale system.**

The Author has showed in [4] that for given set X of cardinality four, concluded Hue, Saturation and Value in the HSV color space and Mean, respectively, one has determined the table of hyper-operations with upper and lower approximation operators of a fuzzy approximation space(X, R), where R is a T-similarity relation on X and T is a continuous triangular norm on unit interval [0, 1]. The best way to model mathematically this decision
function is by considering the basis of the fuzzy set theory. Now, at first we define new fuzzy sets with characteristic functions in the main theorem of Ref. [4] and the following variant operator: \( m_{ad} = 1 - (m_{hi} + m_s + m_v) \) for the gray scale estimation.

Now with aid of graph theory and our defined fuzzy sets in [4], we use MATLAB algorithm for image segmentation and reconstruction. At first, define the vector of data changes, \( c_{ij} \), as the Euclidean distance [12] between the fields (like coordinates, image RGB channels, image grayscale, etc.) on nodes \( v_i \) and \( v_j \), furthermore an indicator vector is used to indicate membership of a node, edge in a set denoted by \( x_i \) and finally, we use for each \( v_i \in S \) volume of \( S \) denoted \( \text{Vol}_S \) defined as follow:

\[
\begin{align*}
    x_i &= \begin{cases} 
        0 & \text{if } v_i \notin S \\
        1 & \text{if } v_i \in S
    \end{cases}
    \quad \text{and } \text{Vol}_S = \sum_i d_i.
\end{align*}
\]

By employing the notion of volume, gives an isoperimetric ratio for an indicator vector, \( x \), as \( h(x) = \frac{x^T L x}{x^T d} \) and \( 0 \leq h(x) \leq 1 \) (to set a threshold and stop parameter) [5].

We consider the following steps to data clustering or image segmentation for the isoperimetric MATLAB algorithm: 

**Step 1:** we calculate weights for all needs edges in the equation: \( w_{ij} = e^{-(t_1 c_1 + t_2 c_2)} \), where \( t_1 \) and \( t_2 \) are scale [5] to normalize the vectors \( c_1 \) and \( c_2 \), then the Laplacian matrix \( L \) is introduced. 

**Step 2:** Choose the node of largest degree as the ground point, \( v_g \), and to compute \( x_0 = d_0 \) by eliminating the row / column corresponding to \( v_g \) in the linear equation \( L x_0 = d_0 \).

**Step 3:** Threshold the potentials \( x \) the value that gives partitions corresponding to the lowest isoperimetric ratio and continue our recursion on each segment until the
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isoperimetric ratio of the subpartitions (i.e. the spectral partitions) is larger than the stop parameter, see our outputs in Figures 3-6.

Figure 5. Comparison of images segmented with pyramid and lattice based isoperimetric algorithms.

Figure 6. Segmentation of image data in a space-variant image.

(a) Original image
(b) Visualization of the space-variant image by our MATLAB algorithm
(c) Visualization of the space-variant image by our color naming algorithm [13, 14]
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References


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