Electron Quantum Transport Through a Mesoscopic Device: Dephasing and Absorption Induced by Interaction with a Complicated Background

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Abstract. Effect of a complicated many-body environment is analyzed on the chaotic motion of a quantum particle in a mesoscopic ballistic structure. The absorption and dephasing phenomena are treated on the same footing in the framework of a schematic microscopic model. The single-particle doorway resonance states excited in the structure via an external channel are damped not only because of the escape onto such channels but also due to ulterior population of the long-lived background states. The transmission through the structure is presented as an incoherent sum of the flow formed by the interfering damped doorway resonances and the retarded flow of the particles re-emitted by the environment.
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1 Introduction

Extensive study of the electron transport through ballistic micro-structures [1] has drawn much attention to peculiarities of chaotic wave interference in open mesoscopic set-ups. It is well recognized by now that the statistical approach [2–4] based on the random matrix theory (RMT) provides a reliable basis for describing the universal fluctuations which, in particular, manifests itself in the single-particle resonance chaotic scattering and the transport phenomena. However experiments with the ballistic quantum dots reveal appreciable and persisting up to very low temperatures deviation from the predictions of the standard RMT, which indicates some loss of the quantum-mechanical coherence. This effect is called dephasing.

Two different methods of accounting for the dephasing have been suggested, which give different results. In the phenomenological Büttiker's voltage-probe model [5] an subsidiary randomizing scatterer has been introduced with M_ϕ channels each with a sticking coefficient T_ϕ. Even assuming all such channels to be statistically equivalent we are still left with two independent parameters. This results in an ambiguity since quantities of physical interest (e.g. the conductance distribution) depend, generally, on M_ϕ and T_ϕ separately whereas the dephasing phenomenon is controlled by the unique parameter: the dephasing time τ_ϕ which is fixed by their product. On the other hand, only one parameter: the strength of a uniform imaginary potential governs the Efetov's model [6] which points at the absorption as the cause of the loss of quantum coherence.

The difference and deficiencies of the two models [7,8] have been analyzed in [8]. A prescription was suggested how to get rid of uncertainties, and simultaneously, to accord the models by considering the limit $M_\phi \to \infty$, $T_\phi \to 0$ at fixed product $M_\phi T_\phi \equiv \Gamma_\phi$ in the first model and by compulsory restoration of the broken because of the absorption unitarity in the second. The construction proposed infers a complicated internal structure of the Büttiker's probe which should, in particular, possess a dense energy spectrum. Otherwise, the assumed limit could hardly be physically justified. If so, the typical time τ_p spent by the scattering particle inside the probe, being proportional to its mean spectral density, forms a new time scale different from the dephasing time. Below we propose a microscopic schematic model of absorption and dephasing phenomena induced by interaction with a very complicated environment with very high density of the energy levels.

2 Chaotic resonance scattering against a complicated background

2.1 Fragmentation of the resonance states because of the influence of the background

Let D be the mean level spacing between the single-particle resonance states in an open cavity with perfectly reflecting walls and no environment. The cavity is supposed to be attached to two similar leads which support M channel states. An open system of such a kind is described by an effective non-Hermitian Hamiltonian $\mathcal{H}^{(s)} = H^{(s)} - \frac{i}{2}AA^\dagger$ where the rectangular matrix A consists of the $M^{(s)}$ column vectors of coupling amplitudes between internal and channel states. Further, let the Hermitian matrix $H^{(e)}$ represent the Hamiltonian of a a many-body environment with a very small mean level spacing δ. The latter fixes the smallest energy scale which will never appear explicitly but ensures unitarity of the scattering matrix $S(E) = I - i\mathcal{T}(E)$. The total system: the cavity interacting with the environment, is described by the extended non-Hermitian effective Hamiltonian

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}^{(s)} & V^\dagger \\ V & H^{(e)} \end{pmatrix}.$$

The corresponding transition matrix equals

$$\mathcal{T}(E) = A^\dagger \mathcal{G}_D(E) A$$

where $\mathcal{G}_D(E)$ stands for the upper left block

$$\mathcal{G}_D(E) = \frac{I}{E - \mathcal{H}^{(s)} - \Sigma(E)}$$

of the resolvent $\mathcal{G}(E) = \frac{I}{E-\mathcal{H}}$ of the extended non-Hermitian Hamiltonian $\mathcal{H}$. The subscript D means "doorway" and marks the states in the ideal cavity, which are directly connected to the scattering channels. In zero approximation $V \equiv 0$, only these states have complex eigenenergies, $\mathcal{E}_n = \varepsilon_n - \frac{i}{2}\Gamma_n$. The environment states get excess to the channels only due to the mixing with the doorway resonances exclusively through which they can be excited or decay. The matrix $\Sigma(E) = V^\dagger \frac{1}{E-H} V$ accounts for transitions cavity $\leftrightarrow$ environment and remains Hermitian (and, correspondingly, the scattering matrix remains unitary) as long as the energy spectrum of the environment is discrete, i.e. the mean level spacing $\delta \neq 0$.

In the single-particle mean field approximation, (quasi)electrons move in the background in a field which is random because of impurities. The dimension $N^{(e)}$ of the Hilbert space of the quasi-particle in the background is much larger than that $N^{(s)}$ of the particle in the mesoscopic cavity, $N^{(e)} \gg N^{(s)}$. Correspondingly, the single-particle mean level spacing d is much smaller than the doorway mean spacing, $d \ll D$, although it is very much larger, $d >>> \delta$, than the many-body level spacing δ. Supposing, therefore, that the coupling matrix elements are random,

$$\langle V_{\mu m} \rangle = 0\,,\ \langle V^*_{\mu m} V_{\nu n} \rangle = \frac{1}{2}\Gamma_s \frac{d}{\pi} \delta_{\mu\nu}\delta_{mn}\,,$$

the matrix Σ reduces after such an averaging to the function

$$\Sigma(E) = \frac{1}{2}\Gamma_s\, g(E); \quad g(E) = \frac{d}{\pi} Tr \frac{1}{E - H^{(e)}}\,.$$

Here the subscripts m, n and μ, ν mark the doorway and the background single-particle states respectively and $\Gamma_s = 2\pi \frac{\langle |V|^2 \rangle}{d}$ is the spreading width which characterizes the scale of the fine-structure fragmentation of the doorway states because of the coupling to the background. The loop function $g(E)$ is real so that the V-averaging does not destroy the unitarity.

The transition amplitudes reduce to the sums of the doorway resonant contributions

$$\mathcal{T}^{ab}(E) = \sum_n \frac{\mathcal{A}^a_n \mathcal{A}^b_n}{E - \mathcal{E}_n - \frac{1}{2}\Gamma_s g(E)} \equiv \sum_n \frac{\mathcal{A}^a_n \mathcal{A}^b_n}{\mathcal{D}_n(E)}\,. \tag{1}$$

For the certainty sake, we restrict ourselves for some time to the case of systems with time reversal symmetry. Then the decay amplitudes A^a_n are real and the matrix of non-Hermitian effective Hamiltonian $\mathcal{H}^{(s)}$ is symmetric. The appearing in Eq. (1) amplitudes $\mathcal{A}^a_n$ are the matrix elements of the coupling matrix $\mathcal{A} = \Psi^T A$ with Ψ being the orthogonal ($\Psi^T\Psi = 1$) matrix of the eigenstates of the effective Hamiltonian $\mathcal{H}^{(s)}$. Unlike the real matrix elements of the matrix A, those of the matrix $\mathcal{A}$ are complex quantities [9].

The exact resonance spectrum $\{\mathcal{E}_\alpha\}$ is found from the equation

$$\mathcal{E} - \mathcal{E}_n - \frac{1}{2}\Gamma_s g(\mathcal{E}) = 0.$$

Each doorway state is fragmented to $\sim \mathit{\Gamma}_s/d$ narrow fine-structure resonances. The enveloping curve has the Lorentzian shape with the width $\mathit{\Gamma}_s$. Finally, transition amplitudes can be represented as coherent sums of interfering contributions of the exact resonances $\mathcal{E}_\alpha$,

$$\mathcal{T}^{ab}(E) = \sum_\alpha \frac{\mathcal{A}^a_\alpha \mathcal{A}^b_\alpha}{E - \mathcal{E}_\alpha} \, .$$

Literally, no dephasing takes place yet. Phases are tuned in such a way that the unitarity condition is fulfilled. However, as distinct from the case of ideal cavity the interference pattern depends now on the two additional parameters: the fine-structure level spacing d and the spreading width $\mathit{\Gamma}_s$.

3 Averaging over the fine-structure scale

Let us suppose that the energy resolution $\mathit{\Delta}E$ is not perfect and do not allow us to resolve the fine structure of the doorway resonances, $d \ll \mathit{\Delta}E$ though $\mathit{\Delta}E \ll D$. Then only averaged cross sections

$$\overline{\sigma^{ab}(E)} = \frac{1}{\mathit{\Delta}E} \int_{E-\frac{1}{2}\mathit{\Delta}E}^{E+\frac{1}{2}\mathit{\Delta}E} dE' \, \sigma^{ab}(E')$$

are measured. To carry out the energy averaging explicitly, we neglect the level fluctuations on the fine structure scale and assume the uniform spectrum, $\varepsilon_\mu = \mu d$ (*picket fence approximation*). This yields immediately $g(E) = \cot\left(\frac{\pi E}{d}\right)$.

3.1 Isolated doorway resonance

In the case of an isolated doorway resonance with the width $\mathit{\Gamma} = \sum_c \mathit{\Gamma}^c \ll D$ though $\mathit{\Gamma} \gg \mathit{\Delta}E$, which is situated at the energy $E_{res} = 0$ the transition cross section equals

$$\sigma^{ab}(E) = \left|\mathcal{T}^{ab}(E)\right|^2 = \frac{\mathit{\Gamma}^a \mathit{\Gamma}^b}{\left[E - \frac{1}{2}\mathit{\Gamma}_s \cot\left(\frac{\pi E}{d}\right)\right]^2 + \frac{1}{4}\mathit{\Gamma}^2}$$

and the energy averaging yields

$$\overline{\sigma^{ab}(E)} = \frac{\mathit{\Gamma}^a \mathit{\Gamma}^b}{E^2 + \frac{1}{4}\left(\mathit{\Gamma} + \mathit{\Gamma}_s\right)^2} + \frac{\mathit{\Gamma}^a \mathit{\Gamma}^b}{\mathit{\Gamma}} \frac{\mathit{\Gamma}_s}{E^2 + \frac{1}{4}\left(\mathit{\Gamma} + \mathit{\Gamma}_s\right)^2} \, .$$

The phase coherence is destroyed by the averaging and the result is given by a sum of two incoherent contributions. The first one corresponds to a single resonance widened because absorption by the environment. This contribution is obtained with the aid of shifting by the distance $\frac{1}{2}\mathit{\Gamma}_s$ in the upper part of the complex energy plane. The second term accounts for the particles reinjected from the background. There is no net loss of particles. All of them are, finally, back. The environment looks from outside as a black box which swallows particles and spits them back after some time.

The transport through the cavity is characterized by the quantity [3,4]

$$G(E) = \sum_{a\in 1, d\in 2} \overline{\sigma^{ab}(E)} = \frac{\Gamma_1\Gamma_2}{\Lambda(E)} + \frac{\Gamma_1\Gamma_2}{\Gamma_1+\Gamma_2}\frac{\Gamma_s}{\Lambda(E)} = T_{12} + \frac{T_{1s}T_{s2}}{T_{1s}+T_{s2}} \tag{2}$$

where $\Lambda(E) = E^2 + \frac{1}{4}\left(\Gamma+\Gamma_s\right)^2$. The second term in the final expression is expressed in the terms of the subsidiary transitions probabilities

$$T_{sk}(E) = \frac{\Gamma_s\,\Gamma_k}{\Lambda(E)}\,, \quad \Gamma_k = \sum_{c\in k}\Gamma^c, \quad k=1,2, \quad \Gamma_1+\Gamma_2 = \Gamma \tag{3}$$

entailed by an additional fictitious channel with the partial width Γ_s. The result (2) is identical to that of the Büttiker's voltage probe model [5] of dephasing phenomenon with the dephasing rate given by $\gamma_s = \frac{2\pi}{D}\Gamma_s = \Gamma_s\tau_D$ where τ_D is the mean delay time in the cavity.

One can simulate such a situation by introducing an additional fictitious $(M^{(s)}+1)$th channel with the transition amplitude $A^{(s)} = \sqrt{\Gamma_s}$ which connects the resonance state to the environment. It is easy to check that the fictitious scattering matrix $\tilde{S}(E) = I - i\tilde{T}(E)$ built in such a way is unitary. Nevertheless, one should remember that the similarity is not perfect. Indeed, in the case of transition from an individual initial channel b onto a subgroup of the final channels the model cross section equals

$$\sum_{c\in sub}\tilde{\sigma}^{cb}(E) = \frac{1+\Gamma_s/\sum_{c\in sub}\Gamma^c}{1+\Gamma_s/\Gamma}\sum_{c\in sub}\overline{\sigma^{cb}(E)} \geq \sum_{c\in sub}\overline{\sigma^{cb}(E)}\,.$$

The equality takes place only if the summation over the final channels is extended to all of them.

The single-particle approximation used above is well justified only when the scattering energy E is close to the Fermi surface in the environment. For higher scattering energies, many-body effects should be taken into account, which result, in particular, in finite lifetime of the quasi-particle. The simplest way to do this is to attribute some imaginary part to quasi-particle's energy, $\varepsilon_\mu = \mu d - \frac{i}{2}\Gamma_e$. Strictly speaking, this suggestion destroys the unitarity of the S-matrix in contradiction with what has been suggested above. However, an initially excited single-particle state with the energy ε_μ evolves because of many-body effects quite similar to a quasi-stationary state till the time $2\pi/\delta$. A particle delayed for such a long time can be considered as irreversibly absorbed.

The resonant denominator in the Eq. (1) equals in this case [10]

$$D_{res}(E) = E - E_{res} - \frac{1}{2}\Gamma_s(1-\xi^2)\frac{\eta}{1+\xi^2\eta^2} + \frac{i}{2}\left(\Gamma + \Gamma_s\xi\frac{1+\eta^2}{1+\xi^2\eta^2}\right)$$

where the following notations have been used:

$$\xi = \tanh\left(\frac{\pi\Gamma_e}{2d}\right), \quad \eta = \cot\left(\frac{\pi E}{d}\right)$$

The total averaged transport cross section $G(E)$ still retains its form (2) but the subsidiary transition probabilities (3) read now as

$$T_{sk}(E) \Rightarrow T_{sk}(E;\kappa) = \frac{\Gamma_s\,\Gamma_k}{\Lambda(E;\kappa)}$$

where

$$\frac{1}{\Lambda(E;\kappa)} = \frac{1}{\Lambda(E)}\,\frac{1}{1+\kappa\frac{\Lambda(E)}{\Gamma\Gamma_s}}\,. \tag{4}$$

The parameter κ is defined as

$$\kappa = \frac{4\xi}{(1-\xi)^2} = e^{\Gamma_e\tau_d} - 1 \approx \begin{cases} \Gamma_e\tau_d\,, & \Gamma_e \ll 1/\tau_d \\ e^{\Gamma_e\tau_d}, & \Gamma_e \gg 1/\tau_d \end{cases}$$

with $\tau_d = 2\pi/d$ being the mean delay time of the particle in the background. The absorption is small if the particle's lifetime $\tau_e = 1/\Gamma_e$ in the environment is noticeably larger than the delay time τ_d. In the opposite case the quasi-particle has enough time to decay and irreversible absorption takes place. The transition probabilities (4) vanish in such a case and our consideration reproduces in this limit the result of Efetov's imaginary potential model [6] with the strength $\alpha = \frac{\pi}{D}\Gamma_s = \frac{1}{2}\gamma_s$. Notice that the crossover from the first to the second regime is very sharp because of the exponential dependence on the dimensionless parameter $\gamma_e \equiv \Gamma_e\tau_d$.

3.2 Overlapping doorway resonances

In the general case of overlapping doorway resonances, the average cross section reads accordingly to the Eq. (1)

$$\overline{\sigma^{ab}(E)} = \sum_{n'n} \mathcal{A}^{a}_{n'}{}^{*}\mathcal{A}^{b}_{n'}{}^{*}\mathcal{A}^{a}_{n}\mathcal{A}^{b}_{n}\,\overline{\frac{1}{\mathcal{D}^{*}_{n'}(E)\mathcal{D}_n(E)}}\,. \tag{5}$$

A bit tedious calculation yields

$$\overline{\frac{1}{\mathcal{D}^{*}_{n'}(E)\mathcal{D}_n(E)}} = \frac{1}{\tilde{\mathcal{D}}^{*}_{n'}(E)\tilde{\mathcal{D}}_n(E)}\left[1 + \frac{\Gamma_s^2}{-i\Gamma_s\left(\mathcal{E}^{*}_{n'}-\mathcal{E}_n\right)+\kappa\,\tilde{\mathcal{D}}^{*}_{n'}(E)\tilde{\mathcal{D}}_n(E)}\right]$$

where $\tilde{\mathcal{D}}_n(E) = E - E_n + \frac{i}{2}\left(\Gamma_n + \Gamma_s\right)$.

Let us suppose first that $\kappa = 0$. Then taking into account the following two identities

$$\frac{i}{\mathcal{E}^*_{n'} - \mathcal{E}_n} = \int_0^\infty dt_r e^{i\left(\mathcal{E}^*_{n'} - \mathcal{E}_n\right)t_r};$$
$$\frac{e^{-i\mathcal{E}_n t_r}}{\tilde{\mathcal{D}}_n(E)} = -ie^{-i\left(E + \frac{i}{2}\Gamma_s\right)t_r} \int_{t_r}^\infty dt\, e^{iEt - i\left(\mathcal{E}_n - \frac{i}{2}\Gamma_s\right)t}$$

we can represent the averaged cross section (5) as a sum, $\overline{\sigma^{ab}(E)} = \sigma_d^{ab}(E) + \sigma_r^{ab}(E)$, of incoherent flows the first of which,

$$\sigma_d^{ab}(E) = \Big|\sum_n \frac{\mathcal{A}_n^a \mathcal{A}_n^b}{\tilde{\mathcal{D}}_n(E)}\Big|^2 \tag{6}$$

describes the contribution of the doorway states damped because of the capture by the environment when the second,

$$\sigma_r^{ab}(E) = \Gamma_s \int_0^\infty dt_r \sigma_r^{ab}(E; t_r), \quad \sigma_r^{ab}(E; t_r) = \Big|\sum_n \frac{\mathcal{A}_n^a \mathcal{A}_n^b}{\tilde{\mathcal{D}}_n(E)}\, e^{-i\mathcal{E}_n t_r}\Big|^2$$

accounts for the particles which spend some time t_r in the background, repopulate the doorway levels, and finally escape from the cavity via the channel a.

With the help of the Bell-Steinberger relation

$$\frac{1}{\mathcal{E}^*_{n'} - \mathcal{E}_n} = -i \frac{U_{n'n}}{\sum_c \mathcal{A}_{n'}^c \mathcal{A}_n^c}$$

contribution $G_r(E) = \sum_{a \in 1, d \in 2} \overline{\sigma_r^{ab}(E)}$ of the re-injected particles to the $1 \to 2$ transmission can be transformed to

$$G_r(E) = \sum_{n'n} U_{n'n} \sqrt{U_{n'n'} U_{nn}} \frac{\sum_{a \in 1} {\Phi_{n'}^a}^* \Phi_n^a \, \sum_{b \in 2} {\Phi_{n'}^b}^* \Phi_n^b}{\sum_{a \in 1} {\Phi_{n'}^a}^* \Phi_n^a + \sum_{b \in 2} {\Phi_{n'}^b}^* \Phi_n^b}\,.$$

Here $U = \Psi^\dagger \Psi$ is the matrix of non-orthogonality of the overlapping doorway states and the subsidiary amplitudes

$$\Phi_n^a(E) = \frac{\sqrt{\Gamma_s}\, \mathcal{A}_n^a / \sqrt{U}_{nn}}{\tilde{\mathcal{D}}_n(E)}$$

implicate the fictitious channel. The quantities $\Gamma^a = \frac{1}{U_{nn}} |\mathcal{A}_n^a|^2$ satisfy the condition $\sum_a \Gamma^a = \Gamma$ and can be interpreted as the partial widths.

In the case of moderately overlapping doorway resonances the matrix $U_{n'n} \approx \delta_{n'n}$ and only probabilities $\left|\Phi_n^a(E)\right|^2$ contribute. Then the result is similar to that of the Büttiker's model. However, the amplitudes $\Phi_n^a(E)$ interfere when the overlap is strong. It is due to the fact that the returning particles cannot escape directly but rather repopulate the doorway states before leaving the cavity.

4 Ensemble averaging

If the particle motion in the cavity is classically chaotic, which is the case of the main interest, the ensemble averaging should be carried out. It can be shown then that such an averaging fully eliminates the spreading width from the mean cross section as long as the many-body effects in the background are fully neglected. Indeed, the ensemble averaged cross section (6) is expressed in the terms of the S-matrix two-point correlation function $C_V^{ab}(\varepsilon) = C_0^{ab}(\varepsilon - i\Gamma_s)$ as

$$\langle \sigma_d^{ab}(E)\rangle = C_V^{ab}(0) = C_0^{ab}(-i\Gamma_s) = \int_0^\infty dt\, e^{-\Gamma_s t}\, K_0^{ab}(t)$$

where the subscript V indicates the coupling to the background and the function $K_0^{ab}(t)$ is the Fourier transform of the correlation function $C_0^{ab}(\varepsilon)$. Using the identity

$$\frac{e^{-i\mathcal{E}_n t_r}}{\tilde{\mathcal{D}}_n(E)} = \frac{1}{2\pi}\, e^{\Gamma_s t_r/2} \int_0^\infty dt\, e^{iEt} \int_{-\infty}^{\infty} dE'\, \frac{e^{-iE'(t+t_r)}}{\tilde{\mathcal{D}}_n(E')}$$

one can convince oneself that

$$\langle \sigma_r^{ab}(E;t_r)\rangle = \int_0^\infty dt\, e^{-\Gamma_s t}\, K_0^{ab}(t+t_r)$$

and, finally,

$$\begin{aligned}\langle \overline{\sigma^{ab}(E)}\rangle &= \textstyle\int_0^\infty dt\, e^{-\Gamma_s t}\, K_0^{ab}(t) + \Gamma_s \int_0^\infty dt_r \int_0^\infty dt\, e^{-\Gamma_s t}\, K_0^{ab}(t+t_r)\\ &= \textstyle\int_0^\infty dt\; K_0^{ab}(t) = C_0^{ab}(0) \equiv \sigma_0^{ab}(E)\,.\end{aligned} \tag{7}$$

Separating in Eq. (5) the part which does not depend of the spreading width after the ensemble averaging we arrive at the conclusion that $\langle \overline{\sigma^{ab}(E)}\rangle = \sigma_0^{ab}(E) + \Delta\sigma_r^{ab}(E)$ where

$$\begin{aligned}\Delta\sigma_r^{ab}(E) &\equiv \kappa \langle \textstyle\sum_{n'n} \mathcal{A}_{n'}^{a\;*} \mathcal{A}_{n'}^{b\;*} \mathcal{A}_n^a \mathcal{A}_n^b\, \frac{1}{\mathcal{E}_{n'}^* - \mathcal{E}_n}\\ &\times \left[\frac{1}{\mathcal{E}_{n'}^* - \mathcal{E}_n}\, \frac{1}{1 + i\frac{\kappa}{\Gamma_s} \tilde{\mathcal{D}}_{n'}^*(E)\tilde{\mathcal{D}}_n(E)/\left(\mathcal{E}_{n'}^* - \mathcal{E}_n\right)}\right]\rangle\end{aligned} \tag{8}$$

In spite of the presence of absorption the cross section (8) can still be expressed in terms of the Fourier transform $K_0^{ab}(t)$ of the two-point correlation function $C_0^{ab}(\varepsilon)$. To do this we first formally expand the expression in the second line into power series with respect to the parameter κ and then make use of the relations:

$$\begin{aligned}\frac{1}{\left(\mathcal{E}_{n'}^* - \mathcal{E}_n\right)^{(k+1)}} &= -\frac{i}{k!} \textstyle\int_0^\infty dt\, e^{\Gamma_s t}\, (-it)^k\, e^{-i\tilde{\mathcal{D}}_{n'}^*(E)t}\, e^{i\tilde{\mathcal{D}}_n(E)t};\\ \tilde{\mathcal{D}}_n^k(E)\, e^{i\tilde{\mathcal{D}}_n(E)t} &= \left(-i\frac{d}{dt}\right)^k\, e^{i\tilde{\mathcal{D}}_n(E)t};\\ e^{i\tilde{\mathcal{D}}_n(E)t} &= -e^{iEt}\frac{1}{2\pi i} \textstyle\int_{-\infty}^{\infty} dE'\, \frac{e^{-iE't}}{\tilde{\mathcal{D}}_n(E')}\,.\end{aligned}$$

The following up summation brings us to the result

$$\Delta\sigma_r^{ab}(E) = -\frac{\kappa}{(2\pi)^2}\int_0^\infty dt_r \int_0^\infty dt\, e^{\Gamma_s(t_r+t)}\left[e^{-\frac{\kappa t}{\Gamma_s}\frac{\partial^2}{\partial t_1 \partial t_2}}\, e^{iE(t_1-t_2)}\right.$$
$$\left.\times \int_0^\infty dE_1 \int_0^\infty dE_2\, e^{iE_1(t_r+t_2)-E_2(t_r+t_1)}\, C_0\left(E_1 - E_2 - i\Gamma_s\right)\right]_{t_1=t_2=t}$$

which after a change of the variables:

$$\overline{E} = \tfrac{1}{2}(E_1+E_2), \qquad \bar{t} = \tfrac{1}{2}(t_1+t_2),$$
$$\varepsilon = E_1 - E_2 \qquad \tau = t_1 - t_2$$

and integration over the variables $\overline{E}$ and ε yields

$$\Delta\sigma_r^{ab}(E) = -\sqrt{\tfrac{\kappa\Gamma_s}{4\pi}}\int_0^\infty dt_r \int_0^\infty \tfrac{dt}{\sqrt{t}}\, e^{-t\left[-\frac{d}{dt_r} - \frac{\kappa}{4\Gamma_s}\left(\frac{d}{dt_r}-\Gamma_s\right)^2\right]}\, K_0^{ab}(t_r)$$
$$= -\sqrt{\tfrac{\kappa\Gamma_s}{4}}\int_0^\infty \frac{dt_r}{\sqrt{-\frac{d}{dt_r} - \frac{\kappa}{4\Gamma_s}\left(\frac{d}{dt_r}-\Gamma_s\right)^2}}\, K_0^{ab}(t_r)\,.$$

(Notice that the form-factors $K_0^{ab}(t)$ monotonously decreases with the time t.) Being presented in such a form this expression is equally valid for both the orthogonal (GOE) as well as the unitary (GUE) symmetry groups.

To simplify subsequent calculation we will consider the case of an appreciably large number $M \gg 1$ of statistically equivalent scattering channels all with the maximal transmission coefficient $T = 1$. Then the channel indices a, b can be dropped and the characteristic decay time of the function $K(t)$ $t_W = 1/\Gamma_W = t_H/M$ is much shorter then the Heisenberg time $t_H = 2\pi/D$ (here $\Gamma_W = \frac{D}{2\pi}M$ is the so called Weisskopf width). It is convenient to represent the function $K_0(t)$, which is real, positive definite and satisfies the conditions $K_0(t<0) = 0,\ K_0(0) = 1$ in the form of the mean-weighted decay exponent [11]

$$K_0(t) = \int_0^\infty d\Gamma\, e^{-\Gamma t}\, w(\Gamma)\,.$$

Rigorously, the weight functions $w(\Gamma)$ are different in the intervals $t < t_H$ and $t > t_H$. However contribution of the latter domain vanishes as e^{-M} [11] when the number of channel grows. Neglecting such a contribution we obtain

$$\Delta\sigma_r^{ab}(E) = -\sqrt{\frac{\kappa\Gamma_s}{4}}\int_0^\infty d\Gamma\, \frac{w(\Gamma)}{\Gamma}\, \frac{1}{\sqrt{\Gamma + \frac{\kappa}{4\Gamma_s}(\Gamma+\Gamma_s)^2}}\,. \tag{9}$$

If the parameter $\kappa \gg \frac{4\Gamma_s\Gamma_W}{(\Gamma_s+\Gamma_W)^2}$ the found expression ceases to depend on it and reduces to

$$\Delta\sigma_r^{ab}(E) \Rightarrow -\Gamma_s \int_0^\infty d\Gamma\, \frac{w(\Gamma)}{\Gamma\,(\Gamma+\Gamma_s)}$$

$$= -\Gamma_s \int_0^\infty dt_r \int_0^\infty dt\, e^{-\Gamma_s t}\, K_0^{ab}(t+t_r)\,.$$

Independently of the symmetry class considered, this contribution perfectly compensates the second term in the r.h.s. of the Eq. (7). The resulting mean cross section is identical to that of the Efetov's imaginary-potential model [6].

On the contrary, in the limit of weak absorbtion $\kappa \ll \frac{4\Gamma_s\Gamma_W}{(\Gamma_s+\Gamma_W)^2}$ the result which reads as

$$\Delta\sigma_r^{ab}(E) \Rightarrow -\sqrt{\frac{\kappa\Gamma_s}{4}}\int_0^\infty d\Gamma\,\frac{w(\Gamma)}{\Gamma^{\frac{3}{2}}} \tag{10}$$

strongly depends on presence or absence of the time-reversal symmetry. In the first case the well known asymptotic expansion [2] of the two-point correlation function yields in the considered case of perfect coupling to the continuum [11]

$$w^{(GOE)}(\Gamma) = \delta(\Gamma-\Gamma_W) - \frac{2}{t_H}\delta'(\Gamma-\Gamma_W) + \frac{M}{2t_H^2}\delta''(\Gamma-\Gamma_W) + ... \tag{11}$$

whereas in the second case

$$w^{(GUE)}(\Gamma) = \delta(\Gamma-\Gamma_W) + ... \tag{12}$$

where contributions of the omitted terms in (8) are $O(1/(M^{-7/2}))$. Within this accuracy we obtain for the transport functions (2):

$$G^{(GOE)}(E) = \tfrac{M_1M_2}{M}\left[\left(1-\sqrt{\tfrac{\kappa\gamma_s}{4M}}\right) - \tfrac{1}{M}\left(1-\tfrac{9}{8}\sqrt{\tfrac{\kappa\gamma_s}{4M}}\right)\right],$$
$$G^{(GUE)}(E) = \tfrac{M_1M_2}{M}\left(1-\sqrt{\tfrac{\kappa\gamma_s}{4M}}\right).$$

The difference

$$\Delta G(E) \equiv G^{(GUE)}(E) - G^{(GOE)}(E) = \frac{M_1M_2}{M^2}\left(1-\frac{9}{8}\sqrt{\frac{\kappa\gamma_s}{4M}}\right) \tag{13}$$

is referred to as the weak localization whereas suppression of this term implies dephasing. We conclude, therefore, that the decay of quasi-particles in the environment not only induces reduction of the total outgoing flow but accounts also for the dephasing effect.

The regime of weak absorption considered above is restricted to the range $0 < \kappa \le \frac{4\Gamma_s\Gamma_W}{(\Gamma_s+\Gamma_W)^2}$ which is very narrow if one out of the two widths Γ_s, Γ_W appreciably exceeds another. Indeed, if for example $\Gamma_s \gg \Gamma_W$ and therefore $\kappa \ll 4\Gamma_W/\Gamma_s \ll 1$ the probability to penetrate into the environment prevails and the particle can finally escape from the cavity through an open channel only if the decay width of quasi-particle in the environment is small enough, $\gamma_e \ll 4\gamma_s/M$. Otherwise absorption dominates. On the contrary, if $\Gamma_W \gg \Gamma_s$ and therefore $\kappa \ll 4\Gamma_s/\Gamma_W = 4\gamma_s/M$ the penetration probability is small and particles mostly leave the cavity before penetrating. In the both cases just mentioned the parameter $\sqrt{\frac{\kappa\gamma_s}{4M}} \ll \frac{1}{\sqrt{M}}$ and the influence of the environment in Eq. (13) is negligible. The discussed range becomes maximal, $0 < \kappa \lesssim 1$, when $\Gamma_s \approx \Gamma_W$. In reality, the spreading width Γ_s which describes relatively weak influence of the environment is expected to be much smaller than the characteristic escape width Γ_W if the number of open channels $M \gg 1$. But the adduced arguments show that the role of the considered mechanism of suppression of the weak localization increases in the case of small number of open channels, which is the most interesting one from the practical point of view. It should be stressed that, rigorously, the asymptotic expansions (11, 12) for the weight functions $w(\Gamma)$ are not justified [11] in the case of few number of channels. However, at least qualitatively, our arguments remain valid.

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