Direct Adaptive Control in Unknown Nonlinear Systems that exhibit Brunovski Canonical Form, using Neuro-Fuzzy High Order Neural Networks, with Robustness Analysis

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***Abstract*— The direct adaptive regulation of nonlinear dynamical systems in Brunovsky form with modeling error effects, is considered in this paper. The method is based on a new Neuro-Fuzzy Dynamical System definition, which uses the concept of Fuzzy Adaptive Systems (FAS) operating in conjunction with High Order Neural Network Functions (HONNFs). Since the plant is considered unknown, we first propose its approximation by a special form of a Brunovsky type fuzzy dynamical system (FDS) assuming also the existence of disturbance expressed as modeling error terms depending on both input and system states. The fuzzy rules are then approximated by appropriate HONNFs. This practically transforms the original unknown system into a neuro-fuzzy model which is of known structure, but contains a number of unknown constant value parameters. The development is combined with a sensitivity analysis of the closed loop in the presence of modeling imperfections and provides a comprehensive and rigorous analysis of the stability properties of the closed loop system. The proposed scheme does not require a-priori information from the expert on the number and type of input variable membership functions making it less vulnerable to initial design assumptions. The existence and boundness of the control signal is always assured by introducing a novel method of parameter hopping and incorporating it in weight updating law. Simulations illustrate the potency of the method and its applicability is tested on the well known benchmarks "Inverted Pendulum" and "Van der pol", where it is shown that our approach is superior to the case of simple Recurrent High Order Neural Networks (RHONNs).**

I. Introduction

Nonlinear dynamical systems can be represented by general nonlinear dynamical equations of the form

$$\dot{x} = f(x, u) \tag{1}$$

The mathematical description of the system is required, so that we are able to control it. Unfortunately, the exact mathematical model of the plant, especially when this is highly nonlinear and complex, is rarely known and thus appropriate identification schemes have to be applied which will provide us with an approximate model of the plant.

It has been established that neural networks and fuzzy inference systems are universal approximators [1], [2], [3],i.e., they can approximate any nonlinear function to any prescribed accuracy provided that sufficient hidden neurons and training data or fuzzy rules are available. Recently, the combination of these two different technologies has given rise to fuzzy neural or neuro fuzzy approaches, that are intended to capture the advantages of both fuzzy logic and neural networks. Numerous works have shown the viability of this approach for system modeling [4] - [12].

The neural and fuzzy approaches are most of the time equivalent, differing between each other mainly in the structure of the approximator chosen. Indeed, in order to bridge the gap between the neural and fuzzy approaches several researchers introduce adaptive schemes using a class of parameterized functions that include both neural networks and fuzzy systems [6] - [12]. Regarding the approximator structure, linear in the parameters approximators are used in [10], [13], and nonlinear in [14], [15], [16].

Adaptive control theory has been an active area of research over the past years [17]-[34]. The identification procedure is an essential part in any control procedure. In the neuro or neuro fuzzy adaptive control two main approaches are followed. In the indirect adaptive control schemes [17] - [19], first the dynamics of the system are identified and then a control input is generated according to the certainty equivalence principle. In the direct adaptive control schemes [20] - [22] the controller is directly estimated and the control input is generated to guarantee stability without knowledge of the system dynamics. Also, many researchers focus on robust adaptive control that guarantees signal boundness in the presence of modeling errors and bounded disturbances [23] - [31]. In [32] both direct and indirect approaches are presented, while in [33],[34] a combined direct and indirect control scheme is used.

Recently [35], [36], high order neural network function approximators (HONNFs) have been proposed for the identification of nonlinear dynamical systems of the form (1), approximated by a Fuzzy Dynamical System. This approximation depends on the fact that fuzzy rules could be identified with the help of HONNFs. The same rationale has been employed in [37], where a neuro-fuzzy approach for the indirect control of unknown systems has been introduced.

In this paper HONNFs are also used for the neuro fuzzy direct control of nonlinear dynamical systems in Brunovsky canonical form with modeling errors. In the proposed approach the underlying fuzzy model is of Mamdnani type. The structure identification is also made off-line based either on human expertise or on gathered data. However [38], the required a-priori information obtained by linguistic information or data is very limited. The only required information is an estimate of the centers of the output fuzzy membership functions. Information on the input variable membership functions and on the underlying fuzzy rules is not necessary because this is automatically estimated by the HONNFs. This way the proposed method is less vulnerable to initial design assumptions. The parameter identification is then easily addressed by HONNFs, based on the linguistic information regarding the structural identification of the output part and from the numerical data obtained from the actual system to be modeled.

We consider that the nonlinear system can be expressed in Brunovsky canonical form. We also consider that its unknown nonlinearities could be approximated with the help of two independent fuzzy subsystems. We also assume the existence of disturbance expressed as modeling error terms depending on both input and system states. Every fuzzy subsystem is approximated from a family of HONNFs, each one being related with a group of fuzzy rules. Weight updating laws are given and we prove that when the structural identification is appropriate and the modeling error terms are within a certain region depending on the input and state values, then the error reaches zero very fast. Also, an appropriate state feedback is constructed to achieve asymptotic regulation of the output, while keeping bounded all signals in the closed loop.

The paper is organized as follows. Section II presents the concept of fuzzy systems (FS) description using rule indicator firing functions and reports on the ability of HONNFs to act as fuzzy rule approximators. The direct neuro fuzzy regulation of dynamical systems in Brunovsky canonical form under the presence of modeling errors is presented in Section III, where the associated weight adaptation laws are given. Simulation results on the control of the well known benchmarks "Inverted Pendulum" and "Van der pol" oscillator with the additional comparisons are given in Section IV, showing that by following the proposed procedure one can obtain asymptotic regulation in a much better way than by just simply using RHONN controllers. Finally, Section V concludes the work.

II. Fuzzy system description using rule firing indicator functions and HONNF

In this section, we are briefly introducing the representation of fuzzy systems using the rule firing indicator functions (RFIF), or simply indicator functions (IF), which is used for the development of the proposed method.

Let us consider the system with input space $u \subset R^m$ and state - space $x \subset R^n$, with its i/o relation being governed by the following equation

$$z(k) = f(x(k), u(k)) \tag{2}$$

where $f(\cdot)$ is a continuous function and k denotes the temporal variable. In case the system is dynamic the above equation could be replaced by the following differential equation

$$\dot{x}(k) = f(x(k), u(k)) \tag{3}$$

By setting $y(k) = [x(k), u(k)]$, Eq. (2) may be rewritten as follows

$$z(k) = f\left(y(k)\right) \tag{4}$$

with $y \subset R^{m+n}$

In case f in (4) is unknown we may wish to approximate it by using a fuzzy representation. In this case both $y(k) = [x(k), u(k)]$ and $z(k)$ are initially replaced by fuzzy linguistic variables. Experts or data depended techniques may determine the form of the membership functions of the fuzzy variables and fuzzy rules will determine the fuzzy relations between $y(k)$ and $u(k)$. Sensor input data, possibly noisy and imprecise, enter the fuzzy system, are fuzzified, are processed by the fuzzy rules and the fuzzy implication engine and are in the sequel defuzzified to produce the estimated $z(k)$ [2], [3]. We assume here that a Mamdani type fuzzy system is used.

Let now $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_{n+m}}$ be defined as the subset of (x,u) pairs, belonging to the $(j_1, j_2, ..., j_{n+m})^{th}$ input fuzzy patch and pointing - through the vector field $f(\cdot)$ - to the subset of $z(k)$, which belong to the $(j_1, j_2, ..., j_{n+m})^{th}$ output fuzzy patch. In other words, $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_{n+m}}$ contains input value pairs that are associated through a fuzzy rule with output values.

According to the above notation the Indicator Function (IF) connected to $\Omega^{l_1,l_2,\ldots,l_n}_{j_1,j_2,\ldots,j_{n+m}}$ is defined as follows:

$$I^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}}(x(k),u(k)) = \begin{cases} \alpha & if\,(x(k),u(k)) \in \Omega^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}} \\ 0 & otherwise \end{cases} \tag{5}$$

where α denotes the firing strength of the rule.

Define now the following system

$$z(k) = \sum \bar{z}^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}} \times I^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}}(x(k),u(k)) \tag{6}$$

Where $\bar{z}^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}} \in R^n$ be any constant vector consisting of the centers of the membership functions of each output variable z_i and $I^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}}(x(k),u(k))$ is the IF. Then, according to [35], [36] the system in (6) is a generator for the fuzzy system (FS).

It is obvious that Eq. (6) can be also valid for dynamic systems. In its dynamical form it becomes

$$\dot{x}(k) = \sum \bar{x}^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}} \times I^{l_1,\ldots,l_n}_{j_1,\ldots,j_{n+m}}(x(k),u(k)) \tag{7}$$

Where $\bar{x}^{l_1,\dots,l_n}_{j_1,\dots,j_{n+m}} \in R^n$ be again any constant vector consisting of the centers of fuzzy partitions of every variable x_i and $I^{l_1,\dots,l_n}_{j_1,\dots,j_{n+m}}(x(k),u(k))$ is the IF.

Based on the fact that functions of high order neurons are capable of approximating discontinuous functions [35] and [36] use high order neural network functions HONNFs in order to approximate the IF $I^{l_1,\dots,l_n}_{j_1,\dots,j_{n+m}}$. A HONNF is defined as:

$$N(x(k),u(k);w,L)=\sum_{hot=1}^{L} w_{hot}\prod_{j\in I_{hot}}\Phi_j^{d_j(hot)} \tag{8}$$

where $I_{hot}=\{I_1,I_2,...,I_L\}$ is a collection of L not-ordered subsets of $\{1,2,...,m+n\}$, $d_j(hot)$ are non-negative integers, Φ_j are sigmoid functions of the state or the input, which are the elements of the following vector

$$\Phi=[\,\Phi_1\ \dots\ \Phi_n\ \Phi_{n+1}\ \dots\ \Phi_{m+n}\,]^T=$$
$$=[\,S(x_1)\ \dots\ S(x_n)\ S(u_1)\ \dots S(u_m)\,]^T \tag{9}$$

where

$$S(x)=a\frac{1}{1+e^{-\beta x}}-\gamma \tag{10}$$

and $w:=[w_1\cdots w_L]^T$ are the HONNF weights. Eq. (8) can also be written

$$N(x(k),u(k);w,L)=\sum_{hot=1}^{L} w_{hot}S_{hot}(x(k),u(k)) \tag{11}$$

where $S_{hot}(x(k),u(k))$ are high order terms of sigmoid functions of the state and/or input.

III. Direct adaptive neuro-fuzzy control

A. Problem formulation and neuro-fuzzy representation

1) Problem formulation: We consider nonlinear dynamical systems of the Brunovski canonical form

$$\dot{x}=A_c x+b_c[f(x)+g(x)\cdot u] \tag{12}$$

where the state $x\in R^n$ is assumed to be completely measured, the control input $u\in R$, f and g are scalar nonlinear functions of the state being only involved in the dynamic equation of x_n. Also, $A_c=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&0&\cdots\\\cdots&\cdots&0&\cdots&0\\0&0&\cdots&0&1\\0&0&\cdots&0&0\end{bmatrix}$ and $b_c=[\,0\ \ \cdots\ \ 0\ \ 1\,]^T$.

The state regulation problem is known as our attempt to force the state to zero from an arbitrary initial value by applying appropriate feedback control to the plant input. However, the problem as it is stated above for the system (12), is very difficult or even impossible to be solved since the f, g are assumed to be completely unknown. To overcome this problem we assume that the unknown plant can be described by the following model arriving from a neuro-fuzzy representation described below, plus a modeling error term $\omega(x,u)$

$$\dot{x}=A_c x+b_c[XW^*S(x)+X_1W_1^*S_1(x)u+\omega(x,u)] \tag{13}$$

where the weight values W^* and W_{1n}^* are unknown.

Therefore, the state regulation problem is analyzed for the system (13) instead of (12). Since, W^* and W_1^* are unknown, our solution consists of designing a control law $u(W,W_1,x)$ and appropriate update laws for W and W_1 to guarantee convergence of the state to zero and in some cases, which will be analyzed in the following sections, boundedness of x and of all signals in the closed loop.

The following mild assumptions are also imposed on (12), to guarantee the existence and uniqueness of solution for any finite initial condition and $u\in U$.

Assumption 1: Given a class U of admissible inputs, then for any $u\in U$ and any finite initial condition, the state trajectories are uniformly bounded for any finite $T>0$. Hence, $|x(T)|<\infty$.

Assumption 2: The *f*, *g* are continuous with respect to their arguments and satisfy a local Lipchitz condition so that the solution $x(t)$ of (12) is unique for any finite initial condition and $u\in U$.

2) Neuro-fuzzy representation: We are using a fuzzy approximation of the system in (12), which uses two fuzzy subsystem blocks for the description of *f*(x) and $g(x)$ as follows

$$f(\chi)=\sum \bar{f}^{l_1,\dots,l_n}_{j_n}\times I^{l_1,\dots,l_n}_{j_n}(\chi) \tag{14}$$

$$g(\chi)=\sum \bar{g}^{l_1,\dots,l_n}_{j_n}\times {I_1}^{l_1,\dots,l_n}_{j_n}(\chi) \tag{15}$$

where the summation is carried out over the number of all available fuzzy rules, *I*, I_1 are appropriate IF and the meaning of indices $\bullet^{l_1,\dots,l_n}_{j_1,\dots,j_n}$ has already been described in Section II.

According to Section II, every IF can be approximated with the help of a suitable HONNF. Therefore, every *I*, I_1 can be replaced with a corresponding HONNF as follows

$$f(\chi)=\sum \bar{f}^{l_1,\dots,l_n}_{j_n}\times N^{l_1,\dots,l_n}_{j_n}(\chi) \tag{16}$$

$$g(\chi)=\sum \bar{g}^{l_1,\dots,l_n}_{j_n}\times {N_1}^{l_1,\dots,l_n}_{j_n}(\chi) \tag{17}$$

where *N*, N_1 are appropriate HONNFs.

In order to simplify the model structure, since some rules result to the same output partition, we could replace the NNs associated to the rules having the same output with one NN and therefore the summations in (16),(17) are carried out over the number of the corresponding output partitions. Therefore, the system of (12) is replaced by the following equivalent Brunovsky form Fuzzy - Recurrent High Order Neural Network (F-RHONN), which depends on the centers of the fuzzy output partitions $\bar{f}_l$ and $\bar{g}_l$

$$\dot{\hat{\chi}}=A_c\hat{\chi}+b_c\left[\sum_{l=1}^{Npf}\bar{f}_l\times N_l(\chi)+\left(\sum_{l=1}^{Npg}\bar{g}_l\times N_{1l}(\chi)\right)u\right] \tag{18}$$

where Npf and Npg are the number of fuzzy partitions of f and g respectively. Or in a more compact form

$$\dot{\hat{\chi}} = A_c\hat{\chi} + b_c[XWS(\chi) + X_1W_1S_1(\chi)u] \qquad (19)$$

where X, X_1 are matrices containing the centers of the partitions of every fuzzy output variable of $f(x)$ and $g(x)$ respectively, $S(\chi), S_1(\chi)$ are matrices containing high order combinations of sigmoid functions of the state χ and W, W_1 are matrices containing respective neural weights according to (11) and (18). The dimensions and the contents of all the above matrices are chosen so that both $XWS(\chi)$ and $X_1W_1S_1(\chi)$ are scalar. For notational simplicity we also assume that all output fuzzy variables are partitioned to the same number, m, of partitions. Under these specifications X is a *1* $\times$ *m* vector of the form

$$X = [\,\bar{f}_1 \quad \bar{f}_2 \quad \cdots \quad \bar{f}_m\,]$$

where $\bar{f}_p$ denotes the center or the fuzzy p-th partition of f. These centers can be determined manually or automatically with the help of a fuzzy c-means clustering algorithm as a part of the off-line structural identification procedure mentioned in the introduction. Also, $S(\chi) = [\,s_1(\chi) \quad \ldots \quad s_k(\chi)\,]^T$, where each $s_i(\chi)$ with $i = \{1, 2, ..., k\}$, is a high order combination of sigmoid functions of the state variables and W is a $m \times k$ matrix with neural weights. W can be also written as a collection of column vectors W^l, that is $W = [\,W^1W^2 \quad \cdots \quad W^l\,]$, where $l = 1, 2, ..., k$. Similarly, X_1 is a $1 \cdot m$ raw vector of the form

$$X_1 = [\,\bar{g}_1 \quad \bar{g}_2 \quad \cdots \quad \bar{g}_m\,],$$

where $\bar{g}_k$ denotes the center or the k-th partition of g. W_1, $S_1(\chi)$ have the same dimensions as W, $S(\chi)$ respectively.

B. Adaptive regulation with modeling error effects

In this subsection we present a solution to the adaptive regulation problem and investigate the modeling error effects when the dynamical equations have the Brunovski canonical form. Assuming the presence of modeling error the unknown system can be written as (13). The regulation of the system can be achieved by selecting the control input to be

$$u = -\frac{XWS(\chi) + v}{X_1W_1S_1(\chi)} \qquad (20)$$

with

$$v = kx \qquad (21)$$

where k is a vector of the form $k = [k_n \cdots k_2\ k_1] \in R^n$ be such that all roots of the polynomial $h(s) = s^n + k_1s^{n-1} + \cdots + k_n$ are in the open left half-plane.

Define now, the regulation error as

$$\xi = -x \qquad (22)$$

After substituting Eq. (20) to the $n - th$ state equation of Eq. (13) and straightforward manipulations we have that

$$\dot{x} = -\Lambda_c x + b_c[X\tilde{W}S(\chi) + X_1\tilde{W}_1S_1(\chi)u - \omega(x,u)] \qquad (23)$$

where $\Lambda_c = A_c - b_ck$ is a matrix with its eigenvalues on the left half plane. $\tilde{W} = W - W^*$ and $\tilde{W}_1 = W_1 - W_1^*$. W and W_1 are estimates of W^* and W_1^* respectively and are obtained by update laws which are to be designed in the sequel. After substituting Eq. (22), (23) becomes

$$\dot{\xi} = \Lambda_c\xi + b_c[X\tilde{W}S(\chi) + X_1\tilde{W}_1S_1(\chi)u - \omega(x,u)] \quad (24)$$

To continue, consider the Lyapunov candidate function

$$V = \frac{1}{2}\xi^T P\xi + \frac{1}{2\gamma_1}tr\left\{\tilde{W}^T\tilde{W}\right\} + \frac{1}{2\gamma_2}tr\left\{\tilde{W}_1^T\tilde{W}_1\right\} \quad (25)$$

Where $P > 0$ is chosen to satisfy the Lyapunov equation

$$P\Lambda_c + \Lambda_c^T P = -I$$

If we take the derivative of Eq. (25) with respect to time we obtain

$$\dot{V} = -\tfrac{1}{2}\left\|\xi\right\|^2 + \\ +\xi^T Pb_cX\tilde{W}S(x) + \xi^T Pb_cX_1\tilde{W}_1S_1(x)u - \\ -\xi^T Pb_c\omega(x,u) + \tfrac{1}{\gamma_1}tr\left\{\dot{\tilde{W}}^T\tilde{W}\right\} + \tfrac{1}{\gamma_2}tr\left\{\dot{\tilde{W}}_1^T\tilde{W}_1\right\}$$

Hence, if we choose

$$tr\left\{\dot{\tilde{W}}^T\tilde{W}\right\} = -\gamma_1\xi^T Pb_cX\tilde{W}S(x) \qquad (26)$$

$$tr\left\{\dot{\tilde{W}}_1^T\tilde{W}_1\right\} = -\gamma_2\xi^T Pb_cX_1\tilde{W}_1S_1(x)u \qquad (27)$$

$\dot{V}$ becomes

$$\dot{V} \le -\frac{1}{2}\left\|\xi\right\|^2 + \left\|\xi\right\|\left\|Pb_c\right\|\left\|\omega(x,u)\right\| \qquad (28)$$

It can be easily verified that Eqs. (26) and (27) after making the appropriate operations, result in the following weight updating laws

$$\dot{W} = -\gamma_1X^Tb_c^TP\xi S^T(x) \qquad (29)$$

$$\dot{W}_1 = -\gamma_2X_1^Tb_c^TP\xi S_1^T(x)u \qquad (30)$$

where ξ is the vector defined in (22), u is a scalar and γ_1, γ_2 are positive constants expressing the learning rates.

The above equations can be also element wise written as:

a) for the elements of W

$$\dot{w}_{jl} = -\gamma_1\bar{f}_jp_n\xi s_l(x) \qquad (31)$$

or equivalently $\dot{W}^l = -\gamma_1(X)^T p_n\xi s_l(x)$ for all $l = 1, 2, ..., k$ and $j = 1, 2, ..., m$. Also, p_n is the last (n^{th}) row of P.

b) for the elements of W_1

$$\dot{w}_{jl} = -\gamma_2\bar{g}_jp_n\xi us_{1l}(x) \qquad (32)$$

or equivalently $\dot{W}_1^l = -\gamma_2(X_1)^T p_n\xi us_{1l}(x)$ for all $l = 1, 2, ..., k$ and $j = 1, 2, ..., m$.

Furthermore, we can make the following assumption.

Assumption 3: The modeling error term satisfies

$$|\,\omega(x,u)\,| \le \ell_1'\,|x| + \ell_1''\,|u|$$

where ℓ_1' and ℓ_1'' are known positive constants.

Also, we can find an *a priori* known constant $\ell_u > 0$, such that

$$|u| \leq \ell_u \, |x|$$

and Assumption 3 becomes equivalent to

$$|\omega\,(x)| \leq \ell_1 \, |x| \tag{33}$$

where

$$\ell_1 = \ell_1' + \ell_1'' \ell_u \tag{34}$$

is a positive constant. Employing Assumption 3, Eq. (28) becomes

$$\dot{V} \leq -\frac{1}{2} \|\xi\|^2 + \ell_1 \|\xi\| \, \|Pb_c\| \, \|x\| \tag{35}$$

since $\|x\| = \|\xi\|$ then Eq. (35) becomes

$\dot{V} \leq -\frac{1}{2} \|\xi\|^2 + \ell_1 \|Pb_c\| \, \|\xi\|^2 =$

$$= -\left(\frac{1}{2} - \ell_1 \|Pb_c\|\right) \|\xi\|^2 \tag{36}$$

Hence, if we chose $\|Pb_c\| < \frac{1}{2\ell_1}$ then the Lapyunov candidate function becomes negative.

We are now ready to prove the following theorem

Theorem 1: The control law (20) and (21) together with the update laws (29) and (30) guarantee the following properties

- $\xi, x, W, W_1 \in L_\infty$
- $\lim_{t\to\infty} \xi(t) = 0, \quad \lim_{t\to\infty} x(t) = 0$
- $\lim_{t\to\infty} \dot{W}(t) = 0, \quad \lim_{t\to\infty} \dot{W}_1(t) = 0$

provided that $\|P\| < \frac{1}{2\ell_1}$ and Assumption 3 is satisfied.

Proof: From Eq. (36) we have that $V \in L_\infty$ which implies $\xi, \tilde{W}, \tilde{W}_1 \in L_\infty$. Furthermore $W = \tilde{W} + W^* \in L_\infty$ and $W_1 = \tilde{W}_1 + {W_1}^* \in L_\infty$. Since $\xi \in L_\infty$ this also implies $x \in L_\infty$. Moreover, since V is a monotone decreasing function of time and bounded from below, the $\lim_{t\to\infty} V(t) = V_\infty$ exists so by integrating $\dot{V}$ from 0 to ∞ we have

$$\int_0^\infty |\xi|^2 dt \leq \frac{1}{\frac{1}{2} - l_1 |Pb_c|} \left[V(0) - V_\infty\right] < \infty$$

which implies that $|\xi| \in L_2$. We also have that

$$\dot{\xi} = \Lambda_c \xi + b_c[X\tilde{W}S(\chi) + X_1\tilde{W}_1 S_1(\chi)u - \omega(x,u)]$$

Hence and since $u, \dot{\xi} \in L_\infty$, the sigmoidals are bounded by definition, $\tilde{W}, \tilde{W}_1 \in L_\infty$ and Assumption 3 hold, so since $\xi \in L_2 \cap L_\infty$ and $\dot{\xi} \in L_\infty$, applying Barbalat's Lemma we conclude that $\lim_{t\to\infty} \xi(t) = 0$.

Now, using the boundedness of $u, S(x), S_1(x)$, x and the convergence of $\xi(t)$ to zero, we have that $\dot{W}, \dot{W}_1$ also converge to zero. Hence we have that

$$\lim_{t\to\infty} x(t) = -\lim_{t\to\infty} \xi(t) = 0$$

Thus,

$$\lim_{t\to\infty} x(t) = 0.$$

■

Remark 1: We can't conclude anything about the convergence of the synaptic weights W and W_1 to their optimum values W^* and W_1^* respectively from the above analysis.

The existence of signal u is associated with conditions which guarantee that $X_1 W_1 S_1 \neq 0$. It can be shown that using appropriate projection method [22], the weight updating laws can be modified so that the existence of the control signal can be assured. However, this development is not presented in this paper due to lack of space. The relevant results along with the ones given here are to be presented shortly in a forthcoming work.

IV. Simulation results

To demonstrate the potency of the proposed scheme we present two simulation results. One of them is the well known benchmark "Inverted Pendulum" and the other "Van der pol" oscillator. Both of them present comparisons of the proposed method with the well established approach on the use of RHONN's [41]. The comparison shows off the regulation superiority of our method under the presence of modeling errors.

A. Inverted Pendulum

Let the well known problem of the control of an inverted pendulum. Its dynamical equations can assume the following Brunovsky canonical form [40]

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\sin x_1 - \frac{mlx_2^2 \cos x_1 \sin x_1}{m_C + m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_C + m}\right)} + \frac{\frac{cos x_1}{m_C + m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_C + m}\right)} u + d$$

$$with \quad d = 5x_2 + 10sin(10x_2) + sin(2u) \tag{37}$$

where $x_1 = \theta$ and $x_2 = \dot{\theta}$ are the angle from the vertical position and the angular velocity respectively. Also, $g = 9.8\ m/s^2$ is the acceleration due to gravity, m_c is the mass of the cart, *m* is the mass of the pole, and *l* is the half-length of the pole. We choose $m_c = 1\ kg$, $m = 0.1\ kg$, and $l = 0.5\ m$ in the following simulation. In this case we also have that $|x_1| \leq \pi/6$ and $|x_2| \leq \pi/6$.

It is our intention to compare the direct control abilities of the proposed Neuro-Fuzzy approach with RHONN's [41] (page 62-71). We also, make the appropriate changes to RHONN's in order to be equivalent with F-HONNF's (brunovski form) for comparison purposes.

For the proposed F-HONNF approach we use the adaptive laws which are described by Eqs. (29) and (30) and the control law described by Eqs. (20) and (21). Numerical training data were obtained by using Eq. (37) with initial conditions $[\,x_1(0) \quad x_2(0)\,] = [\,-\frac{\pi}{12} \quad 0\,]$ and sampling time 10^{-3} sec.

We are using the proposed approach with Eq. (19) to approximate Inverted Pendulum dynamics and send data to sigmoidal terms. Our Neuro-Fuzzy model was chosen to use 5 output partitions of *f* and 5 output partitions of *g*. The number of high order sigmoidal terms (HOST) used in

HONNF's (for F-HONNF and RHONN) were chosen to be 5 $(s(x_1), s(x_2), s(x_1) \cdot s(x_2), s^2(x_1), s^2(x_2))$.

In order our model to be equivalent with RHONN's regarding to other parameters except the initial weights ($W(0) = 0$ and $W_1(0) = 1$ for FHONNF and $W(0) = 0$ and $W_1(0) = 0.016$ for RHONN) we have chosen the updating learning rates $\gamma_1 = 5$ and $\gamma_2 = 1$ and the parameters of the sigmoidal terms such as $a_1 = 0.1$, $a_2 = 3$, $b_1 = b_2 = 1$ and $c_1 = c_2 = 0$. Also, $k_1 = 2$ and $k_2 = 80$ after careful selection. Fig. (1) shows the regulation of states x_1 (with blue line for F-HONNF corresponding approach and red line for RHONN's) and x_2 while fig. (2) gives the evolution of control input u, the errors e_1, e_2 and the modeling error $d(x_2, u)$ respectively.

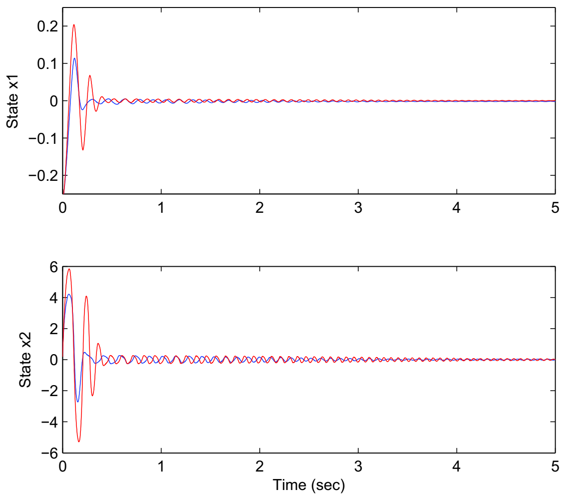

Fig. 1. Evolution of state variable x_1 and x_2 for RHONN's (red line) and F-HONNF approach (blue line)

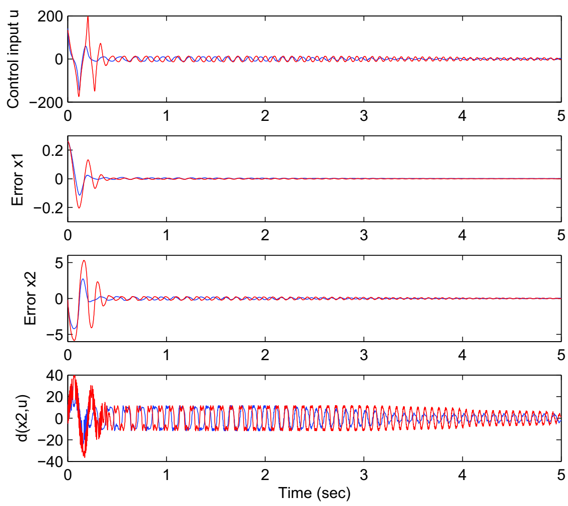

Fig. 2. Evolution of control input u and disturbance d for RHONN's (red line) and F-HONNF approach (blue line)

The mean squared error (MSE) for RHONN and F-HONNF approaches were measured and are shown in Table I, demonstrating a significant (order of magnitude) increase in the regulation performance of F-HONNF against RHONN's.

B. Van der pol

Van der Pol oscillator is usually used as a simple benchmark problem for testing control schemes. It's dynamical equations are given by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_2 \cdot \left(a - x_1^2\right) \cdot b - x_1 + u + d$$
$$with \quad d = 2x_2 + 4sin(5x_2) + 5sin(10u) \tag{38}$$

The procedure of the approximation was the same as that of Inverted Pendulum.

The proposed Neuro-Fuzzy model was chosen to have initial conditions, $[\,x_1(0) \quad x_2(0)\,] = [\,0.4 \quad 0.5\,]$ and the number of high order sigmoidal terms (HOST) used in HONNF's (for F-HONNF and RHONN) were chosen to be 2 $(s(x_1), s(x_2)$. In order our model to be equivalent with RHONN's regarding to other parameters except the initial weights ($W(0) = 0$ and $W_1(0) = 0.1$ for FHONNF and $W(0) = 0$ and $W_1(0) = 0.05$ for RHONN) we have chosen the updating learning rates $\gamma_1 = 0.1$ and $\gamma_2 = 10$ and the parameters of the sigmoidal terms such as $a_1 = 0.1$, $a_2 = 4$, $b_1 = b_2 = 1$ and $c_1 = c_2 = 0$. Also, $k_1 = 2$ and $k_2 = 80$ after careful selection. Fig. (3) shows the regulation of states x_1 and x_2, while fig. (4) presents the evolution of control input u, the errors e_1, e_2 and modeling error d.

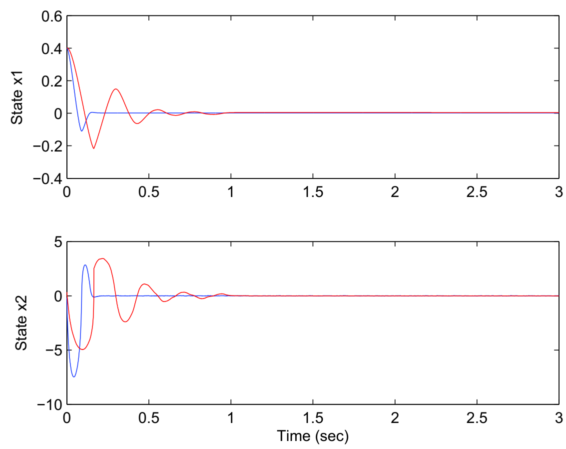

Fig. 3. Evolution of state variable x_1 and x_2 for RHONN's (red line) and F-HONNF approach (blue line)

The mean squared error (MSE) for RHONN and F-HONNF approaches were measured and are shown in Table I demonstrating as before (Inverted Pendulum case), a significant (order of magnitude) increase in the control performance.

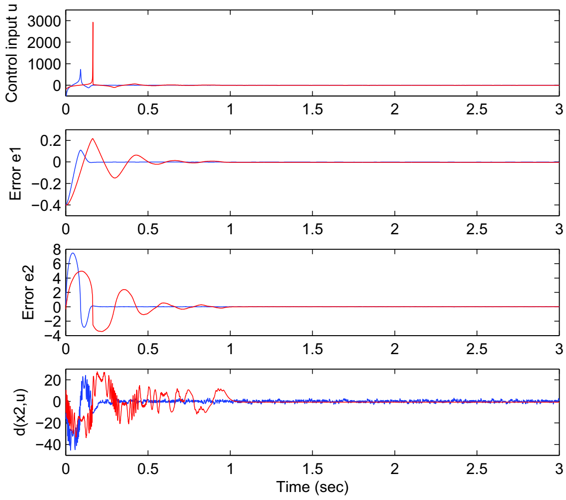

Fig. 4. Evolution of control input u and disturbance d for RHONN's (red line) and F-HONNF approach (blue line)

TABLE I

COMPARISON OF RHONN'S AND F-HONNF APPROACHES FOR THE INVERTED PENDULUM AND VAN DER POL OSCILLATOR.

Examples		RHONN	FHONNF
Inverted Pendulum	$MSEx_1$	0.0010	$6.0794 \cdot 10^{-4}$
	$MSEx_2$	0.9252	0.3368
Van der pol	$MSEx_1$	0.0024	$9.5226 \cdot 10^{-4}$
	$MSEx_2$	0.8417	0.5930

Conclusively, the comparison between RHONN and F-HONNF's leads to a superiority of F-HONNF's regarding the control abilities. Especially, if we choose smaller learning rate and reduce the number of high order terms, then the difference between the two methods is huge.

V. CONCLUSIONS

A direct adaptive control scheme was considered in this paper, aiming at the regulation of non linear unknown plants of Brunovsky canonical form with the presence of modeling errors. The approach is based on a new Neuro-Fuzzy Dynamical Systems definition, which uses the concept of Fuzzy Adaptive Systems (FAS) operating in conjunction with High Order Neural Network Functions (F-HONNFs). Since the plant is considered unknown, we first propose its approximation by a special form of a Brunovsky type fuzzy dynamical system (FDS) where the fuzzy rules are approximated by appropriate HONNFs. This practically transforms the original unknown system into a neuro-fuzzy model which is of known structure, but contains a number of unknown constant value parameters known as synaptic weights. The proposed scheme does not require a-priori experts' information on the number and type of input variable membership functions making it less vulnerable to initial design assumptions. Weight updating laws for the involved HONNFs are provided, which guarantee that the system states reach zero exponentially fast, while keeping all signals in the closed loop bounded. Simulations illustrate the potency of the method while the applicability is tested on well known benchmarks where it is shown that by following the proposed procedure one can obtain asymptotic regulation quite well. Compared to simple RHONN's direct control, proves to be superior.

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