Modelling Complex Multicellular Tumour–Immune Systems

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Abstract: The report develops a variety of mathematical tools to model the dynamics of large systems of interacting cells. Interactions are ruled not only by laws of classical mechanics but also by some biological functions. The mathematical approach is the one of kinetic theory and non-equilibrium statistical mechanics.
Keywords: Cancer modelling, Multiscale modelling, Complexity in biology, Living system, Kinetic theory, Multicellular system, Asymptotic limit.

1. Introduction

The report deals with critical analysis and developments related to literature on multiscale modelling of multicellular systems involving tumour–immune cells competition at the cellular level [1]. Analysis is focused on development of mathematical methods of classical kinetic theory to model the physical system and to recover a macroscopic equation from microscopic description. The aim is organizing various contributions in literature into mathematical framework suitable to generate a theory for complex biological systems.

2. Computational Method

The report develops a variety of mathematical tools to model the dynamics of large systems of interacting cells [2]. Interactions are ruled not only by laws of classical mechanics but also by some biological functions. The mathematical approach is the one of kinetic theory and non-equilibrium statistical mechanics. The report deals with both the derivation of evolution equations and the design of specific models consistent with the mathematical framework. Various hints for research perspectives are proposed.

3. Calculation Results and Discussion

The report deals with the development of new paradigms, based on the methods of the mathematical kinetic theory for active particles, to model the dynamics of large systems of interacting cells [3]. Interactions are ruled not only by laws of classical mechanics, but also by a few biological functions, which are able to modify the laws above. The report shows how the theory can be applied to model large complex systems in biology. The last part of the report deals with a critical analysis and with the indication of research perspectives concerning the challenging target of developing a biological-mathematical theory for the living matter. The report deals with the analysis of the asymptotic limit towards the derivation of hyperbolic macroscopic equations for a class of equations modelling complex multicellular systems

[4]. Cellular interactions generate both modification of biological functions and proliferating destructive events related to growth of tumour cells in competition with the immune system (Figure 1) [5]. The asymptotic analysis refers to the hyperbolic limit to show how the macroscopic tissue behaviour can be described by linear and nonlinear hyperbolic systems, which seems the most natural in this context (Figure 2).

From the present results and discussion the following conclusions can be drawn. (1) Selected topics related to the modelling of cancer onset, evolution and growth are presented. (2) Tumour spheroids grown *in vitro* have been widely used as models of *in vivo* tumour growth.

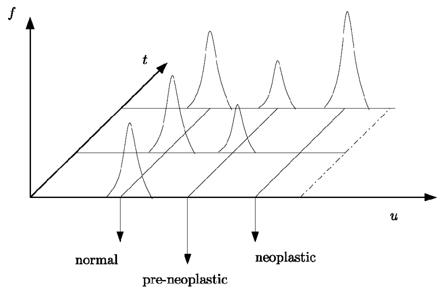

Figure 1. Heterogeneity and progression of tumour cells: Time evolution of the probability distribution (*f*) over the variable (*u*) expressing the differentiation state of cells

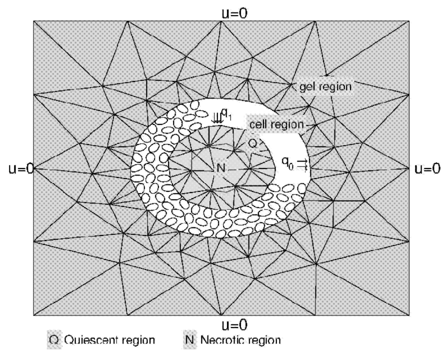

Figure 2. Computational grid for tumour speroid growth, with a continuum necrotic core (N), quiescent region (Q) and surrounding gel, along with a discrete proliferating cell region (white)

References

[1] N. Bellomo, A. Bellouquid and M. Delitala. Mathematical topics on the modelling complex multicellular systems and tumor immune cells competition. *Math. Models Meth. Appl. Sci*., 14:1683, 2004.

[2] A. Bellouquid and M. Delitala. Mathematical methods and tools of kinetic theory towards modelling complex biological systems. *Math. Models Meth. Appl. Sci*., 15:1639, 2005.

[3] N. Bellomo and G. Forni. Looking for new paradigms towards a biological-mathematical theory of complex multicallular systems. *Math. Models Meth. Appl. Sci*., 16:1001, 2006.

[4] N. Bellomo, A. Bellouquid, J. Nieto and J. Soler. Multicellular biological growing systems: Hyperbolic limits towards macroscopic description. *Math. Models Meth. Appl. Sci*., 17:S1675, 2007.

[5] N. Bellomo, N.K. Li and P.K. Maini. On the foundations of cancer modelling: selected topics, speculations, and perspectives. *Math. Models Meth. Appl. Sci*., 18:593, 2008.