System Dynamical Rebuilding Interacting with Environment During its Evolution

(plenary report)

Vilor L. ZAKOVOROTNY (Professor)

Don State Engineering University
1, Gagarin sq., Rostov-on-Don, RUSSIA
E-mail: anatoly.kolesnikov@gmail.com

Abstract: we propose math. model of system complicated evolution. System parameters are varying by extraction of power in system-environment coupling. Parameters are introduced by second kind Volterra integral operator in respect of job-power trajectory. We present methodologies of synthesis for stationary evolution trajectories, as well problems of system dynamical rebuilding and bifurcation.

Introduction

Most of the technical systems exist in terms of interacting of separate elements of theirs through the different environments: aerodynamical, hydrodynamical, tribological, technological etc. Meantime, the dissipation formations, which influence the trajectories of the navigated motions, are formed in the border areas of conjugation of some elements of the system with the environment. The features of these formations, which characterize the dynamic connection been formed, possess the evolutional variability. While its' evolution depends on the trajectories of working and power of irreversible transformations, taking place in the areas of conjugations of the system elements with environment. There are a lot of examples of how the evolutional transformations of environmental characteristics influence the navigated motion trajectories. Studying the evolution of the technical system is traditional and corresponds with the problem of ensuring the reliability. In particular studying the evolutional transformations while processing on the cutting machines affects the development of wearing of the instrument, changing the products quality indexes while cutting etc. [1-3]. But all these indexes characterize the external display of evolutional changes. The evolutional changes of dynamic characteristic of the cutting process, which influences the parameters of the geometrical quality of the goods and the state of the cutting process, perform the more important meaning. They affect the current characteristics of the irreversible transformations while cutting, which are characterized by trajectories of changing the power of irreversible transformations of the function of work being performed. In the technical systems with the friction areas the evolutional changing of the matrixes of the dynamic rigidity and dissipation of tribo-space, being formed in the areas of conjugation of the contacted elements, takes place [4].

Mathematical model of the evolutional transformations in the dynamic environment

While creating the dynamic models of the managed systems the hierarchy principle is used. It's based on separating the motions onto "slow" motions of the executive elements and "rapid" motions, which are considered in the variations towards the trajectories of the "slow" motions [5, 6, 7]. Let's consider as set the trajectories of the "slow" motions of the executive elements, which are being formed in the independent reading system $X = \{X_1, X_2, X_3, X_4\}^T \in \mathbf{X}$. Here and hereafter symbol $\{\cdots\}^T$ means the operation of transposition. For example, in conformity with turning processing vector X components have the following meaning: X_1 - coordinate of the cross-moving support; X_2 is the coordinate of support moving towards the cutting speed (it's obvious, that in the traditional version of the machine $X_2 \equiv 0$); X_3 - coordinate of the support cross-moving; X_4 - angle coordinate of the spindle position. If while processing the frequency of spindle rotation is kept constant, $dX_4 / dt = const$. As follows, the trajectories of executive machine motions are set in the **X** space. If not consider the errors of gears of executive motions, the trajectories, set as phase trajectories, are the program of CNC. For example for a turning machine while programming the trajectory of support motion the speed of supply at settled direction is being set, i.e. phase trajectory $\{X_3, dX_3 / dt\}$. As the result of trajectories of the **X** space being crossed with the detail, positioned in this space, the interactions with the cutting instrument are formed, which are joined with variety of the processes, which in total do determine the processing and the indexes of geometrical quality of the goods. One of the most important of them are cutting forces $F = (F_1(t), F_2(t), F_3(t))^T \in \mathbf{F}$, settled with their own projections. These forces keep working while some of their power. Just the cutting forces work while some of their power leads to changing of properties of the cutting process, causing the evolutional transformations of the dynamic cutting system.

Alike there is a task of managing the system, which has the friction area. The evolutional transformations in the tribo system are characterized by changes of parameters of equations of the ties, which is formed in the friction area. In hydrodynamical system the lifting hydrodynamical force depends not only on speed gradients on equivalent gap in contacted through liquid elements, but also on work and power of irreversible transformations in conjugation areas. In one's turn on lifting power system motion resistance and determinacy of coordinates of state of the managed system do depend . Should underline, that motion is defined not only by management, but also by motion resistance. Further we'll pay more attention to cutting processing. This is to notice the details of the visual effects. It's obvious that general methodics and the way of explanation are alike and for modeling of the other technical systems evolution.

Cutting forces depend on elastic tool deformations towards the part. That is why let's add the vectors of elastic tool deformations $Y(t) = \{Y_1(t), Y_2(t), Y_3(t)\}^T \in \mathbf{Y}$ towards the part $Z(t) = \{Z_1(t), Z_2(t), Z_3(t)\}^T \in \mathbf{Z}$. As follows, in general case the cutting forces are the functions of all coordinates of **X**, **Y** and **Z** spaces. As already noticed, within this article the trajectories of executive machine elements and geometry of the part in the **X** space are considered to be already settled. Following by [5-7], let's define the equation of dynamics of the system, which should also contain the evolutionary changes of parameters $p = \{p_1, p_2, \cdots p_s\}$ of dynamical characteristic of the cutting process (in general case – environment).

$$\begin{cases} m\dfrac{d^2Y(t)}{dt^2} + h\dfrac{dY(t)}{dt} + cY(t) = F(X, \dfrac{dX}{dt}, Y, \dfrac{dY}{dt}, Z, \dfrac{dZ}{dt}, p); \\ M\dfrac{d^2Z(t)}{dt^2} + H\dfrac{dZ(t)}{dt} + CZ(t) = F(X, \dfrac{dX}{dt}, Y, \dfrac{dY}{dt}, Z, \dfrac{dZ}{dt}, p); \\ p^{(i)}(A) = p_{i,0} + p_i \int\limits_0^A w_{P_i}(A-\tau)N(\tau)d\tau, i = 1,2...s; \\ A(t) = \int\limits_0^t N(t)dt; \\ N(t) = V_p\left|F_2(t)\right|. \end{cases} \tag{1}$$

where m, M are diagonal matrixes sized $3\otimes 3$; $c = \lfloor c_{s,k} \rfloor$, $C = \lfloor C_{s,k} \rfloor$ - positively defined symmetrical matrixes of rigidity of subsystem of the instrument and part sized $3\otimes 3$, non-changeable at coordinates of support motions and non-changeable while changing the position of the point of system equilibrium; $h = \lfloor h_{s,k} \rfloor$, $H = \lfloor H_{s,k} \rfloor$ - positively defined symmetrical matrixes of dissipation of the subsystem instrument and part sized $3\otimes 3$, also non-changeable at coordinates of support motions and non-changeable while changing the position of the point of system equilibrium; $w_{P_i}(A-\tau) = \exp[-\dfrac{1}{T_{P_i}}(A-\tau)]$ - kernels of the integral operators, while T_{P_i} - constant works, which are estimated in kGm, these parameters characterize the evolutional trajectories heredity while working. So, the evolutional heredity is displayed only within the integral operator kernel attenuation while working motion is negative (takes into consideration only the pre-history of power on work done); V_P - cutting speed, which in our case is considered constant.

There are preliminary remarks regarding model (1). In the considered system the parameters of the dynamic characteristic of the cutting process have initial values $p_{i,0}$ and p_i values, which depend on the trajectory of work and power. In its' turn the trajectory of work and power is the function of the coordinates of the system state. Further we'll consider the most important case, when work and power do change so slowly, that within a step of integrating the system (1) they can be considered unchangeable. In other words, lagging while counting A and N for one step does not influence the general dynamics of the system. More than that, system (1) contains the parameters of the dynamic model of subsystems of the instrument and part. Methods of counting of these parameters and of their identification are known [5]. As for dynamic models of the cutting process, which expose the relation of the coordinates of executive elements of the machine and elastic deformations of the instrument towards the part, models like this have been considered earlier [8, 9]. In particular it's demonstrated, that around the fixed trajectory, which is settled by "slow" motions of executive elements their linearized performance is possible. Then around the fixed trajectory of the "slow" motions the reaction of the cutting process is determined by matrixes of dynamic rigidity and dissipation of the processing process. While studying the evolutional reorganization of the dynamic system the principal meaning the parameters of kernels of the integrated operators $w_{P_i}(A-\tau)$, which in this case are considered fixed, have. The issues of identification of the kernels will be considered further. Will consider them settled to expose the general principles of analysis.

Features of evolutional transformations of the dynamic system.

In the beginning let`s observe the system, which possesses next features:

- cutting forces satisfy the hypothesis about their constant orientation in space.

This orientation is set by angular coefficients $\chi = (\chi_1, \chi_2, \chi_3)$;

- in variations concerning stationary trajectory, which is assigned by X, the dynamic characteristic can be linearized while the reaction from the side of processing can be replaced by matrixes of the dynamic rigidity and dissipation. Then instead of (1) we have:

$$\begin{cases} m\dfrac{d^2Y(t)}{dt^2}+h\dfrac{dY(t)}{dt}+cY(t)=F(t)\{\chi_1,\chi_2,\chi_3\}; \\ M\dfrac{d^2Z(t)}{dt^2}+H\dfrac{dZ(t)}{dt}+CZ(t)=F(t)\{\chi_1,\chi_2,\chi_3\}; \\ F(t)=F_0+F^{(\Im)}(t)+\alpha(A)(-Y_1(t)-Z_1(t))+\beta(A)(-\dfrac{dY_1}{dt}(t)-\dfrac{dZ_1}{dt}(t)); \\ F^{(\Im)}(t)=f\int\limits_0^A w_\alpha(A-\tau)N(\tau)d\tau; \\ \alpha(A)=\alpha_0+\gamma\int\limits_0^A w_\alpha(A-\tau)N(\tau)d\tau;\beta(A)=\beta_0+\eta\int\limits_0^A w_\beta(A-\tau)N(\tau)d\tau; \\ A(t)=\int\limits_0^t N(t)dt;N(t)=V_p\left|\chi_2F(t)\right|, \end{cases} \quad (2)$$

where f, γ, η - balanced coefficients, which have accordingly the dimension $[\frac{s}{kGm^2}]$, $[\frac{s}{kGm^3}]$ and $[\frac{s^2}{kGm^3}]$, these coefficients characterize intensity of evolutional changes of constant component of cutting force, rigidity and dissipation of processing. In this case evolutional components of constant component of cutting force, rigidity coefficient and dissipation are accordingly measured in $[kG]$, $[kG/m]$ and $[kGs/m]$. For example, if $\eta=0$, then matrixes of dynamic dissipation of cutting process during functioning of the system do not change. In this case the dynamic modification of system is caused firstly by changes of general powered loading of cutting process, secondly – by matrix`s changes of dynamic rigidity of cutting process.

In the system (2) initial conditions are determined by the values of elastic tool deformations $Y^*=\{Y_1^*,Y_2^*,Y_3^*\}^T$ concerning detail $Z^*=\{Z_1^*,Z_2^*,Z_3^*\}^T$ at initial stage of process, when evolutional changes of parameters of dynamic rigidity and dissipation are absent, i. e. $\alpha(A)=\beta(A)=0$. Thus,

$$Y(0)=Y^*=\{Y_1^*,Y_2^*,Y_3^*\}^T,\frac{dY}{dt}(0)=\{0,0,0\}^T,$$
$$Z(0)=Z^*=\{Z_1^*,Z_2^*,Z_3^*\}^T,\frac{dZ}{dt}(0)=\{0,0,0\}^T. \quad (3)$$

In given model the speed of cutting is considered to be constant value. Thus, if system has the point of equilibrium, which is asymptotical steady, the work is made only by force`s component, directed by cutting speed. The coordinates $Y(0)=Y^*$ and $Z(0)=Z^*$, which determine the initial condition of system, are the results of systems

$$cY(0)=F(0),CZ(0)=F(0),F(0)=\{\chi_1F(0),\chi_2F(0),\chi_3F(0)\}^T \quad (4)$$

Functions Y and Z characterize the displacement of a point of equilibrium, caused by features of powered interaction at initial stage of a cutting process and evolutional transformations of system (2) due to work, which is made at some power of irreversible transformations, depended on trajectory. Start of evolutional transformations is at point $t = 0$, where finished work equals zero. That`s why during analysis it's reasonable to examine composition $Y(A) = Y^* + Y^{(E)}(A)$ and $Z(A) = Z^* + Z^{(E)}(A)$. This composition corresponded in time with the other one $Y(t) = Y^* + Y^{(E)}(t)$ and $Z(t) = Z^* + Z^{(E)}(t)$. As work, time, current forces and motions in (2) are interconnected, we may also observe displacement functions $Y(X_3) = Y^* + Y^{(E)}(X_3)$ and $Z(X_3) = Z^* + Z^{(E)}(X_3)$ of the point of equilibrium by moving of instrument relatively to work piece. Here X_3 - is the way, which is made by instrument towards the work piece during manufacturing set of products.

Speaking in general about processing, we should stress that evolution of parameters of dynamic characteristic of cutting process causes the displacement of stationary varieties' parameters, which are formed around the point of equilibrium, causing the dynamic modification of system. Moreover, in some moments of evolutional transformations changes of topology of phase space of subsystem of "fast" movements take place, i.e. bifurcations of system, which essentially change properties of cutting process.

Function $A(t)$ increases at all interval of integration, as subintegral expression is non-negative value at any moment of time. As in resulted simplified model of matrix summarized rigidity and dissipation matrixes have coefficients, which "slowly" change in course of evolutional transformations, not only point of equilibrium is evolutional. Dynamic properties of system for "fast" movements also change. That`s why roots trajectories of characteristic polynomial in complex plane should correspond with evolutional trajectory $Y^{(E)}(t)$ and $Z^{(E)}(t)$. However at high intensity of evolutional transformations the situation when evolutional transformations themselves may influence the dynamic of system is possible. The intensity of evolutional transformations in (2) is determined by coefficients γ and η.

Let`s give an example of the displacement of balanced point of system for shaft turning, parameters of work-piece dynamic model are constant. It`s important to note, that the displacement of balanced point corresponds with a change of work-piece`s diameter. Besides we`ll consider dynamical characteristics of tool subsystem to be constant, as all processing conditions, except values of dynamic rigidity and dissipation of cutting process. The main parameters are listed in tables 1, 2, initial values of rigidity and dissipation accordingly equal $\alpha_0 = 100 kG/mm$ and $\beta_0 = 10 \frac{kGs}{mm}$. Cutting force at initial stage - $F_0 = 100 kG$. Evolutional hereditary constants T_α and

T_β accordingly equal $T_\alpha = 50kGm$ and $T_\beta = 20kGm$. Force orientation coefficients: $\chi_1 = 0{,}50$; $\chi_2 = 0{,}71$; $\chi_3 = 0{,}50$.

We see, that depending on parameters of intensity of evolutional transformations γ and η, displacement curves of a balanced point in direction, normal to work-piece rotation axis, change (fig. 1) and remind tool wear curves in some of their values. This specifies again at relation of tool wear value with changes of dynamical characteristic of cutting process.

Table 1. Tool subsystem parameters

$m, \dfrac{kGs^2}{mm} \cdot 10^{-3}$	$h, \dfrac{kGs}{mm}$	$c, \dfrac{kG}{mm} \cdot 10^3$
$\begin{bmatrix} 0{,}25; & 0; & 0 \\ 0; & 0{,}25; & 0 \\ 0; & 0; & 0{,}25 \end{bmatrix}$	$\begin{bmatrix} 0{,}6; & 0{,}1; & 0{,}08 \\ 0{,}1; & 0{,}5; & 0{,}2 \\ 0{,}08; & 0{,}2; & 0{,}7 \end{bmatrix}$	$\begin{bmatrix} 1{,}0; & 0{,}3; & 0{,}2 \\ 0{,}3; & 1{,}2; & 0{,}4 \\ 0{,}2; & 0{,}4; & 1{,}6 \end{bmatrix}$

Table 2. Work-piece subsystem parameters

$m, \dfrac{kGs^2}{mm} \cdot 10^{-3}$	$h, \dfrac{kGs}{mm}$	$c, \dfrac{kG}{mm} \cdot 10^3$
$\begin{bmatrix} 5{,}0; & 0; & 0 \\ 0; & 5{,}0; & 0 \\ 0; & 0; & 5{,}0 \end{bmatrix}$	$\begin{bmatrix} 5{,}0; & 1{,}0; & 0{,}8 \\ 1{,}0; & 4{,}0; & 0{,}6 \\ 0{,}8; & 0{,}6; & 2{,}0 \end{bmatrix}$	$\begin{bmatrix} 0{,}2; & 0{,}1; & 0{,}08 \\ 0{,}1; & 0{,}2; & 0{,}01 \\ 0{,}08; & 0{,}01; & 1{,}0 \end{bmatrix}$

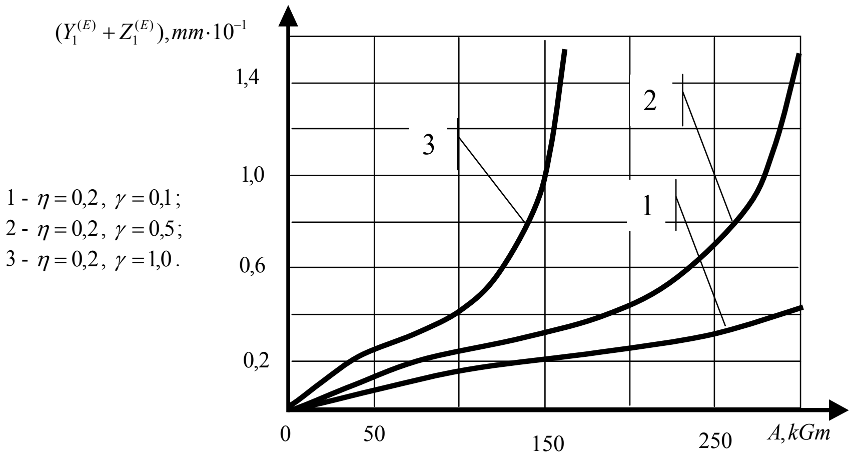
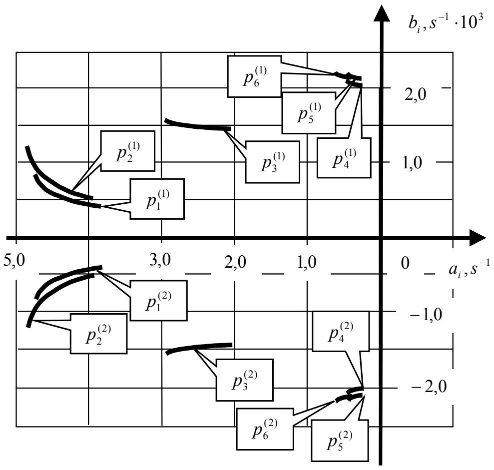


Fig. 1. Displacement of system point of equilibrium in direction, normal to work-piece rotation axis, with it`s evolutional transformations

It`s important to note, that in present time management precision processing practice, based on corresponding subadjustments of machines performed elements coordinates, management is realized with part direct diameter measurement after processing. As a rule, it`s difficult to fulfill this measurement, because an additional operation is required. Given model allows to forecast these modifications in easily measurable trajectories of conversed machine system. Besides evolutional trajectory becomes natural trajectory of system, which is necessary to be included into general management system. Let`s examine root trajectories of characteristic polynomial (fig.2 "a"), which correspond with deviation evolutional trajectories of a balanced point, which are shown on fig. 1. Trajectories are calculated for system characteristic polynomial (2) assuming, that current parameters of system are frozen. This system has six pairs of complex-conjugated roots $p_i^{(1)} = -a_i^{(1)} + jb_i^{(1)}$ and $p_i^{(2)} = -a_i^{(1)} - jb_i^{(1)}$ ($i = 1,2,\cdots 6$), which are located at left complex half-plane, because frozen system is steady at all observed functioning segment. Evolutional displacement of balanced point corresponds with evolutional trajectory of every root or its' material and imaginary components. On fig. 2 "b" an example of separate evolutional stages division by classes with indication of balanced point is shown. Estimation of $Y^{(E)}(t)$ and $Z^{(E)}(t)$ may be absolutely exact, because an observed system is determined and evolutional root**s** curves have no ambiguity sections. But the question about evolutional displacement estimation may arise based on autoregressive spectral analysis with the observation at vibrating succession.



«a»

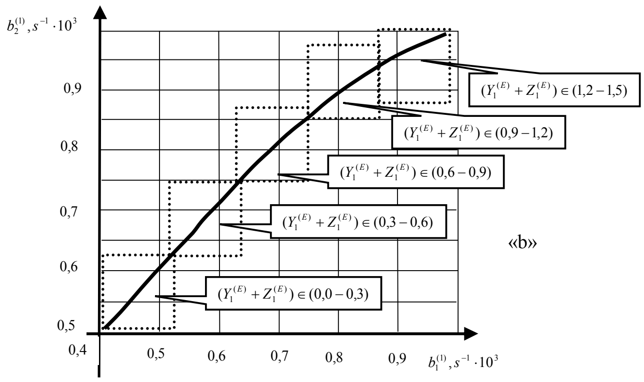


Fig. 2. Evolutional roots trajectories of characteristic polynomial “a” and example of division by classes with the value of evolutional displacement of point of equilibrium “b”

In this case an informational space may be formed, which consisted of material and imaginary roots components. Classification methods, which were earlier examined also may be used, for example at the base of Baies classification rule [9]. In this example we accept the hypothesis about invariability of cutting forces orientation in space, which is correct only in low-frequency field and at wear small variations of cutting tool. While examining the system (1) in variations, we should notice that during evolution redistribution between force components, operated in different directions, is observed. Moreover in summarized rigidity and dissipation matrixes, not only skew-symmetric components are formed, but also matrixes themselves may become negatively determined.

In this case the point of equilibrium may become unstable. After this, in condition space some varieties are formed, which we examined earlier in detail [8, 9]. In this case system properties, including indexes of products geometrical quality, become dependent on initial conditions and small indignations, which influence the system. It`s important to note, that because of skew-symmetric rigidity and dissipation matrixes influence at performed work and power during periodic tool movements relatively to workpiece, variable components of cutting forces, which have difficult orientation in space, start working. The work is made by force components, caused by skew-symmetric terms of rigidity matrix and symmetric terms of dissipation matrix. In this case the work is examined along trajectory of periodical tool movements towards the workpiece.

In that way, instead of (3) we have the following system:

$$\begin{cases} m\dfrac{d^2X(t)}{dt^2} + h\dfrac{dX(t)}{dt} + cX(t) = F(X,\dfrac{dX(t)}{dt}); \\ F(X,\dfrac{dX(t)}{dt}) = \{F_1(X,\dfrac{dX(t)}{dt}), F_2(X,\dfrac{dX(t)}{dt}), F_3(X,\dfrac{dX(t)}{dt})\}^T; \\ F_i(X,\dfrac{dX(t)}{dt}) = F_{0,i} + \alpha_i(-X_1(t)) + \beta_i(-\dfrac{dX_1}{dt}) + \phi_i(-\dfrac{dX_2}{dt}) + \mu_i(-\dfrac{dX_3}{dt}), i = 1,2,3; \\ \alpha_i(A) = \alpha_{0,i} + \alpha_i\int\limits_0^A w_{\alpha,i}(A-\tau)N(\tau)d\tau; \beta_i(A) = \beta_{0,i} + \beta_i\int\limits_0^A w_{\beta,i}(A-\tau)N(\tau)d\tau; \\ \phi_i(A) = \phi_{0,i} + \phi_i\int\limits_0^A w_{\phi,i}(A-\tau)N(\tau)d\tau; \mu_i(A) = \mu_{0,i} + \mu_i\int\limits_0^A w_{\phi,i}(A-\tau)N(\tau)d\tau; \\ A(t) = \int\limits_0^t N(t)dt; N_i(t) = V_p\left|F_2(t)\right|. \end{cases} \quad (5)$$

Operator kernels in (5) have the same structure as in (2). But performed work corresponds (2) only in case, when system has asymptotically steady point of equilibrium. In general case comparatively from (2) work and power are determined for forces, which have different projections at axes X_1, X_2 and X_3. Expressions for calculation of work in general case are carefully analyzed in our monography [4]. Matrix m in system (5) is diagonal. Concerning elasticity $c^{(R)}$ and dissipation $h^{(R)}$ matrixes, rigidity matrix, taking into account reaction from the side of cutting process, equals:

$$c^{(R)} = \begin{bmatrix} \alpha_1(A) = \alpha_{0,1} + \gamma_1\int\limits_0^A w_{\alpha,1}(A-\tau)N(\tau)d\tau; & 0; & 0 \\ \alpha_2(A) = \alpha_{0,2} + \gamma_2\int\limits_0^A w_{\alpha,2}(A-\tau)N(\tau)d\tau; & 0; & 0 \\ \alpha_3(A) = \alpha_{0,3} + \gamma_3\int\limits_0^A w_{\alpha,3}(A-\tau)N(\tau)d\tau; & 0; & 0 \end{bmatrix} \quad (6)$$

And dissipation matrix of cutting process equals:

$$h^{(R)} = \begin{bmatrix} \beta_1(A) = \beta_{0,1} + \beta_1\int\limits_0^A w_{\beta,1}(A-\tau)N(\tau)d\tau; & \phi_1(A) = \phi_{0,1} + \phi_1\int\limits_0^A w_{\phi,1}(A-\tau)N(\tau)d\tau; & \mu_1(A) = \mu_{0,1} + \mu_1\int\limits_0^A w_{\mu,1}(A-\tau)N(\tau)d\tau \\ \beta_2(A) = \beta_{0,2} + \beta_2\int\limits_0^A w_{\beta,2}(A-\tau)N(\tau)d\tau; & \beta_2(A) = \beta_{0,2} + \beta_2\int\limits_0^A w_{\beta,2}(A-\tau)N(\tau)d\tau; & \mu_2(A) = \mu_{0,2} + \mu_2\int\limits_0^A w_{\mu,2}(A-\tau)N(\tau)d\tau \\ \beta_3(A) = \beta_{0,3} + \beta_3\int\limits_0^A w_{\beta,3}(A-\tau)N(\tau)d\tau; & \phi_3(A) = \phi_{0,3} + \phi_3\int\limits_0^A w_{\phi,3}(A-\tau)N(\tau)d\tau; & \mu_3(A) = \mu_{0,3} + \mu_3\int\limits_0^A w_{\mu,3}(A-\tau)N(\tau)d\tau \end{bmatrix}.$$

(7)

In such a way, during evolution we observe elements redistribution of rigidity matrix and dissipation of cutting system. Besides in rigidity matrixes, as a rule, those components, which form force reactions from the cutting process side in directions X_1 and X_3, do increase. Coefficients of dissipation matrix vary as a result of next factors action:

- essentually they depend on value of delayed argument with forming of cutting forces modifications. Delayed argument at constant cutting speed grows during the volume increasing of plastic deformation in cutting area, involved into system reorganization. That`s why dissipation matrixes at small variations of condition coordinates relatively to the balanced point may be negative and as a rule they increase on module during evolutional modifications;

- because of kinetic characteristic of a cutting process at fallen section of forces' dependence on speed, an effect of negative friction in the field of tool front side contact with shaving and in a field, which adjoin to rear surface of an instrument is observed. As a rule this effect increases according to wear-in and wearing development, i.e. during evolution of system.

Let`s examine the sample of evolutional modification of a cutting system in supposition, that dissipation matrixes are constant, i.e. $\beta_i = \phi_i = \mu_i = 0$. We examine only case, when the work, which **is** made while system vibrations around the point of equilibrium **is** negligible small in comparison with the work which is made by the main component of a cutting force as far as moving of tool towards the workpiece. This is fair in those cases when balanced point is asymptotically steady or varieties, which are formed in vicinity of this point, have amplitudes, which are much smaller than the value of allowance on processing. Then instead of (5) we should examine next simplified evolutional cutting system:

$$\begin{cases} m\dfrac{d^2X(t)}{dt^2} + h_\Sigma \dfrac{dX(t)}{dt} + cX(t) = F(X, \dfrac{dX(t)}{dt}); \\ F(X, \dfrac{dX(t)}{dt}) = \{F_1(X, \dfrac{dX(t)}{dt}), F_2(X, \dfrac{dX(t)}{dt}), F_3(X, \dfrac{dX(t)}{dt})\}^T; \\ F_i(X, \dfrac{dX(t)}{dt}) = F_{0,i} + \alpha_i(A)(-X_1(t)), i = 1,2,3; \\ \alpha_i(A) = \alpha_{0,i} + \alpha_i \int\limits_0^A w_{\alpha,i}(A-\tau)N(\tau)d\tau; \\ A(t) = \int\limits_0^t N(t)dt; \\ N_i(t) = V_p |F_2(t)|, \end{cases} \qquad (8)$$

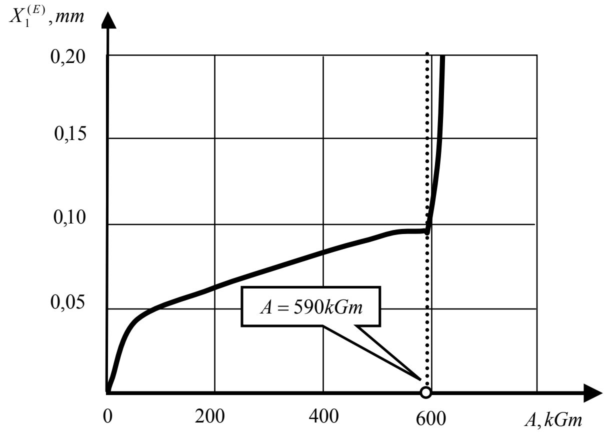
where $h_\Sigma = \begin{bmatrix} (h_{1,1} + \beta_{0,1}); & (h_{2,1} + \phi_{0,1}); & (h_{3,1} + \mu_{0,1}) \\ (h_{1,2} + \beta_{0,2}); & (h_{2,2} + \phi_{0,2}); & (h_{3,2} + \mu_{0,2}) \\ (h_{1,3} + \beta_{0,3}); & (h_{2,3} + \phi_{0,3}); & (h_{3,3} + \mu_{0,3}) \end{bmatrix}$ - summarized dissipation matrix with account of reaction from the side of cutting process.

Let`s examine a concrete example for this case. Main parameters of a system are listed in table 3. Summarized dissipation matrix is conditionally accepted to be symmetrical and positively definite. Initial forces values, determined by technological regimes ($t_R = 2{,}5mm$, $S_R = 0{,}2mm/r$, $V_R = 120{,}0m/\min$) while steel turning 20X by cutting plates of T15K6 are equal to $F_{0,1} = 40{,}0kG$, $F_{0,2} = 100{,}0kG$, $F_{0,3} = 60{,}0kG$. Initial conditions of rigidity of cutting process coefficients are accordingly equal to $\alpha_{0,1} = 100kG/mm$, $\alpha_{0,2} = 80,0kG/mm$, $\alpha_{0,3} = 20,0kG/mm$.

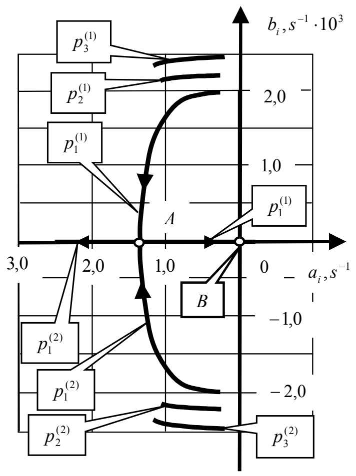
Table 3. **Parameters of subsystem of the instrument with regard of cutting process feedback**

$m, \frac{kGs^2}{mm} \cdot 10^{-3}$	$h_\Sigma, \frac{kGs}{mm}$	$c, \frac{kG}{mm} \cdot 10^3$
$\begin{bmatrix} 0,25; & 0; & 0 \\ 0; & 0,25; & 0 \\ 0; & 0; & 0,25 \end{bmatrix}$	$\begin{bmatrix} 1,2; & 0,2; & 0,1 \\ 0,2; & 1,5; & 0,2 \\ 0,1; & 0,2; & 1,4 \end{bmatrix}$	$\begin{bmatrix} 1,0; & 0,5; & 0,2 \\ 0,5; & 1,2; & 0,4 \\ 0,2; & 0,4; & 1,6 \end{bmatrix}$

As earlier let's study the evolutional diagrams (fig. 3) of displacement of the point of equilibrium of the system $X(t) = X^* + X^{(E)}(t)$ and respective diagrams of displacement of the roots of characteristical polynomial while the initial values of the system (8) are equal $X(0) = \{0,06mm, 0,12mm, 0,09mm\}^T$; $\frac{dX}{dt}(0) = \{0,0,0\}^T$.



«a»



«b»

Fig. 3. Evolutional trajectories of displacement of the point of equilibrium of the system towards X_1 «a» and the roots of characteristical polynomial «b» for the system (8)

We can see that a couple of roots of characteristical polynomial after some evolution become equal to each other and material (point «A» on fig. 3 «b»). After that the roots take different directions on material axis and one of the roots crosses the imaginary axis. At the moment “frozen” system looses the steadiness of the point of equilibrium and X_1 coordinate according to the law of unstable exponent leaves to eternity. So called instrument “undermining” takes place. In that case the whole system endures the double bifurcation transformations. Three laid on each other steady focuses at the initial stage of evolution are being transformed in the point «A» on fig. 3 into two steady focuses, corresponded with vibrations of the relative knot, which asymptotically strives towards the point of equilibrium. Then in the point one of the trajectories becomes unstable, but all other trajectories strive towards it. The example proves that evolutional transformation while the cutting process are not only the instrument wear and (or) changes of the current values of diameter of the processed detail. These are only two external displays of the evolutional system transformations. In our opinion changing of the topology of the phase space, which appears while bifurcation transformations and related changes of the dynamic cutting process characteristic is worth more attention.

One more sample of the evolutional transformations of the linearized system. Let’s consider the evolutional transformations to have only the dissipation matrixes as $\alpha_i \equiv 0$, but $\beta_i \neq 0$, $\phi_i \neq 0$, $\mu_i \neq 0$. As follows, let the described above system has the evolutional transformations coefficients’ matrix correspond to table 4.

Table 4. **The values of coefficients of the evolutional transformations of the cutting process dissipation matrix**

i	$\beta_i, \frac{s}{m^2}$	$\phi_i, \frac{s}{m^2}$	$\mu_i, \frac{s}{m^2}$
$i=1$	-0,02	0,001	0,01
$i=2$	0,01	0,002	0,005
$i=3$	-0,02	0,002	0,005

Let's study the sample of changing the evolutional trajectory of the point of equilibrium, calculated for the case (fig. 4). As was shown earlier, the elements of the matrix of dynamic dissipation of the cutting process around the point of equilibrium can have the negative values. The reasons of the effect are of two kinds. Firstly, the negative values of some coefficients can be caused by lagging argument in the functions of force transformations while varying the coordinates, which affect the forces. Secondly, the negative value while the cutting speed is settled can be caused by kinetic characteristic of the friction speed in the contact area of shaving and the front surface of the instrument and in the contact area of the rear side and the detail. Both these factors increase the affecting while the wear grows, so during the evolutional transformations of the cutting system.

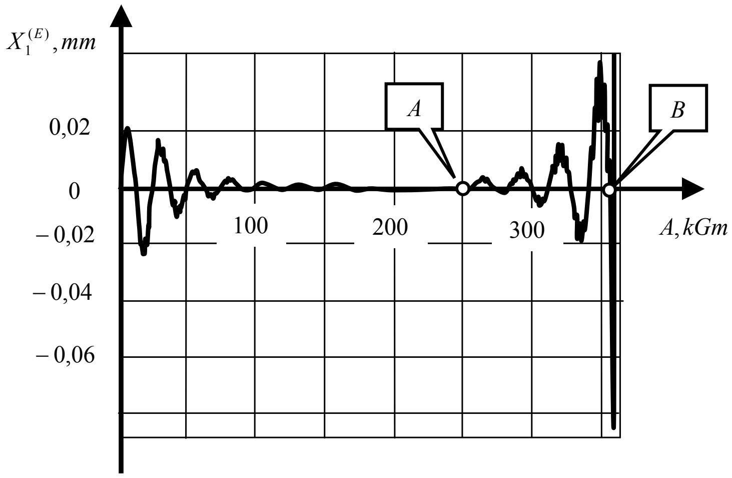


Fig. 4. Evolutional diagram of displacement of the point of equilibrium by changing the dissipation matrixes

One noticeable point is – super low-frequency periodical displacements of the point of equilibrium, which go out slowly and the current values of size do set on some permanent level. So, the evolutional transformations themselves, which are modeled as the integral operators in this case, cause the behavior of the system if the system is considered been "frozen", i.e. the dynamics of the evolutional transformations affects the permanent trajectory, which can be stable or non-stable. In the highlighted sample the permanent mode is set before the work of $(250-260)kGm$ (point

«A» on fig. 4) is done. Further these vibrations arise and starting from the point «B» the frozen dynamic cutting system looses the steadiness of the point of equilibrium. At the same time the couple of complex-conjugated roots of the characteristical polynomial of the frozen system leaves for the right complex semi-surface (not shown on fig.) It's significant, that even small variations β_i, ϕ_i, μ_i can lead to serious dynamic reconstruction of the system. And as the last time the topology of the phase space of the cutting system changes.

While the couple of roots of characteristical polynomial leave for the right complex semi-surface, linearized models become unacceptable. While the amplitude of periodical motions increases within the system the extra-relations, which limit the development of periodical movements are being formed. That is why it's necessary to note the non-linear dynamic models. Here let's study the sample, when non-linear characteristic of the cutting process is performed as had been suggested by Relei:

$$F^{(D)}(t) = \beta_1 v - \beta_2 v^3, \qquad (9)$$

where $v = \frac{dY_1}{dt} + \frac{dZ_1}{dt}$. So, let's take only the case, when the variations of displacements of the instrument towards the part only at X_1 coordinate cause the changes of the major force component, which is separated into its' own projections by matrix of angle coefficients in space.

That's why let's have a look at system (2) again. Total displacements in X_1 direction are determined as $Y_1 + Z_1$. But despite of (2) let's estimate the dynamic characteristic of the cutting process on vibration speeds of the instrument towards the part in non-linear view. Also will consider the initial values of displacements Y_1 and Z_1 equal to zero, so that the system state coordinates we'll consider in the variations towards the static set of the cutting instrument in the independent counting system of the metal cutting machine. So the evolutional equation of dynamics can be performed as

$$\begin{cases} m\frac{d^2Y(t)}{dt^2} + h\frac{dY(t)}{dt} + cY(t) = F(t)\{\chi_1, \chi_2, \chi_3\}; \\ M\frac{d^2Z(t)}{dt^2} + H\frac{dZ(t)}{dt} + CZ(t) = F(t)\{\chi_1, \chi_2, \chi_3\}; \\ F(t) = F_0 + \alpha(A)(-Y_1(t) - Z_1(t)) + \beta_1(A)(\frac{dY_1}{dt}(t) + \frac{dZ_1}{dt}(t)) - \beta_2(A)(\frac{dY_1}{dt}(t) + \frac{dZ_1}{dt}(t))^3; \\ \alpha(A) = \alpha_0 + \alpha_1\int_0^A w_\alpha(A-\tau)N(\tau)d\tau; \beta_1(A) = \beta_{0,1} + \beta_1\int_0^A w_\beta(A-\tau)N(\tau)d\tau; \\ \beta_2(A) = \beta_{0,2} + \beta_2\int_0^A w_\beta(A-\tau)N(\tau)d\tau; \\ A(t) = \int_0^t N(t)dt; N(t) = V_p|\chi_2 F(t)|, \end{cases} \qquad (10)$$

where $\beta_{0,2}$ - non-linear dissipation coefficient, measured as $\frac{kGs^3}{m^3}$; β_2 - non-linear coefficient of the evolutional transformations, measured as $\frac{s^3}{m^4}$.

Initial values (10) are settled

$$Y(0)=\begin{pmatrix}0\\0\\0\end{pmatrix};\ \frac{dY}{dt}(0)=\begin{pmatrix}0\\0\\0\end{pmatrix};\ Z(0)=\begin{pmatrix}0\\0\\0\end{pmatrix};\ \frac{dZ}{dt}(0)=\begin{pmatrix}0\\0\\0\end{pmatrix}. \quad (11)$$

Let's analyze the time and phase trajectories of system motion from the point (11). It's suitable to estimate them as the projection for two phase surfaces $Y, dY/dt$ and $Z, dZ/dt$, as well as in the coordinates system, where the time value is on the abscises axis (fig. 5). So, the phase surface, located to the left of abscises axis, belongs to $Y, dY/dt$, but to the right of the axis – to $Z, dZ/dt$ (fig. 5 «b»). System parameters correspond with the sample mentioned earlier.

We can see, that while $\alpha(A)=0$ after the transformation process the stationary state is being settled with the coordinates $Z_1 = Z_1^* = 0{,}44mm = const$ and $Y_1 = Y_1^* = 0{,}11mm = const$ (fig. 5 «a»). But with the system moving because of increasing $\beta_1(A)$ the roots of characteristical polynomial of the "frozen" system also move in point «A» on fig. 5 «a» crossing the imaginable axis. And around the $Z_1 = Z_1^* = 0{,}44mm = const$ points and $Y_1 = Y_1^* = 0{,}11mm = const$ dissipation matrix of the "frozen" system become negatively defined. That is why the points of equilibrium loose their steadiness and the steady limit cycle, parameters of which do change during the evolution, is being formed around the points. So, the «A» points on fig. 5 «a» are the points of bifurcation of Andronov-Khopf. This is the bifurcation of appearing orbital asymptotically steady limit cycle from asymptotically steady point of equilibrium. If in this case to take into consideration the non-linearity in positioning relation inside the force system function, than with raising amplitude of the periodical motions the dynamic displacing of the Y_1^* and Z_1^* points will be noted. So, the varieties, formed in the space of "fast" motions (in our case these are the parameters of the limit cycle), affect the form generating trajectories of the "slow" motions, as follows – on current values of diameter of the being processed roller.

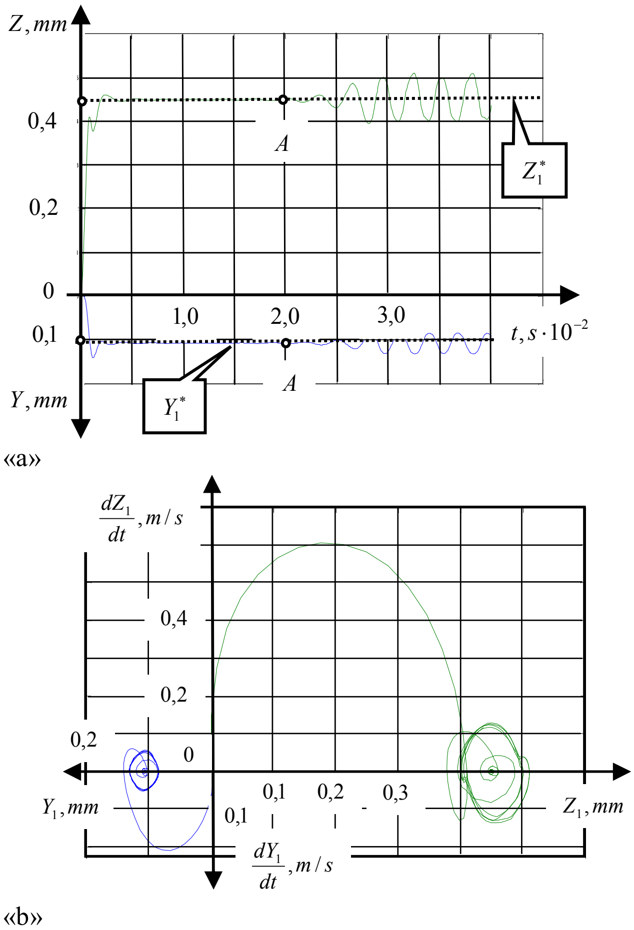


Fig. 5. Sample of evolutional diagram of instrument and detail «a» displacement for system (10) and its' phase trajectories «b» for $\alpha(A) = 0$

Identification of the integral operators' kernels

While constructing the evolutional equations the identification of the integral operators' kernels is the most complex. All integral operators in equation systems are stationary. That is why it's possible to estimate them while wear-in of the cutting instrument and settlement of the stationary state. The general identification algorithm was studied earlier [5]. Let's study the easy way to estimate the kernels of the integral operators on example of defining the changes of dynamic rigidity of the cutting process, when we use as rigidity its' traditional scalar performance, which is accepted in machine-building technology. Let's also consider, that the stationary state of the system is steady. Should note, that constructing the evolutional equations, which allow to estimate the disposition of the point of equilibrium, i.e. changing the detail diameter, is worth working out when producing the details of low rigidity or in cases, when rigidity of subsystem of the instrument is low (e.g. while chiseling the inlets with chiseling arbors). In this case the evolutional equation will be

$$\begin{cases} c_1 Y_1(t) = \chi_1 \{F_0 + [\alpha_0 + \alpha_1 \int\limits_0^A w_\alpha (A-\tau) N(\tau) d\tau](-Y_1(t) - Z_1(t))\}; \\ C_1 Z_1(t) = \chi_1 \{F_0 + [\alpha_0 + \alpha_1 \int\limits_0^A w_\alpha (A-\tau) N(\tau) d\tau](-Y_1(t) - Z_1(t))\}; \\ A(t) = \int\limits_0^t N(t) dt; N(t) = V_p |\chi_2 F(t)|, \end{cases} \qquad (12)$$

where α_0, α_1 are parameters to be identified; $w_\alpha(A-\tau) = \exp\{-\frac{1}{T_\alpha}(A-\tau)\}$ - the kernel of the operator, inside of which it's also necessary to identify the major parameter T_α.

While researching the given parameters of the system (12) are c_1, C_1, χ_1, χ_2 and V_P. And they are constant. The coordinates to estimate are $Y_1(t)$ and $Z_1(t)$. $Y_1(t)$ coordinate is being estimated while direct measuring the elastic instrument deformations, for example with strain-measuring sensors. $Z_1(t)$ coordinate is being determined while measuring the changing of diameter of the detail after processing ($Y_1(t)$ coordinate is known). The force F_0 in the system, for which the following correlation is workable $\alpha_0 / c_1 = \alpha_0 / C_1 \approx 0$, can be measured with defining the elastic deformations. Also due to the meaning of the system (12) when $t = 0$ $\alpha_1 \int\limits_0^A w_\alpha (A-\tau) N(\tau) d\tau\}(-Y_1(t) - Z_1(t)) \equiv 0$. That is why α_0 definition can be realized at the initial stage of the process after the transitional period, which is connected with cutting in the instrument into the detail according to the expression

$$\alpha_0 = \frac{C_1}{C_1 + c_1}[F_0 - \frac{c_1}{\chi_1} Y(0)]. \qquad (13)$$

As the $Y_1(t)$ и $V_P = const$ can be measured, the phase trajectory $\{A, dA/dt = N\}$ can be calculated and performed as vectors $\{A(0), A(1), \cdots A(k)\}^T$ and $\{N(0), N(1), \cdots N(k)\}^T$. Also it's correct of (12) that

$$\alpha_{\Sigma}(A) = \alpha_1 \int_0^A w(A-\xi) N(\xi) d\xi \,, \tag{14}$$

where $\alpha_{\Sigma}(A) = \dfrac{F_0}{Y_1(A) + Z_1(A)} - \dfrac{c_1 Y_1(A) + C_1 Z_1(A)}{2\chi_1 [Y_1(A) + Z_1(A)]} - \alpha_0$.

It's significant, that with $A = 0$ $\alpha_{\Sigma} = 0$, because on this case $2\chi_1 F_0 = c_1 Y_1(0) + C_1 Z_1(0) + 2\chi_1 \alpha_0 [Y_1(0) + Z_1(0)]$ - condition of static balance of the system at initial stage of cutting process. That's why the meaning of the function $\alpha_{\Sigma}(A)$ characterizes changes of the dynamic rigidity of the cutting process caused by work of irreversible transformations done. As the integral operator is stationary and its' parameters can be determined as $w_{\alpha}(A-\tau) = \exp\{-\frac{1}{T_{\alpha}}(A-\tau)\}$, the following functional can be formed

$$I = \sum_{i=0}^{i=k} \{\alpha_{\Sigma}(A_i) - \alpha_1 \int_0^{A_i} [\exp(-\frac{A-\xi}{T_{\alpha}})] N(\xi) d\xi\}^2 = \min \,. \tag{15}$$

It allows to determine α_1 and T_{α} most accurately. It's suitable to choose α_1 and T_{α}, which meet the functional (15) requirements, with Gauss – Zaidel method.

It's possible to simplify the measuring procedure of α_1 and T_{α} further, if take into consideration mathematical expectation of cutting power with index

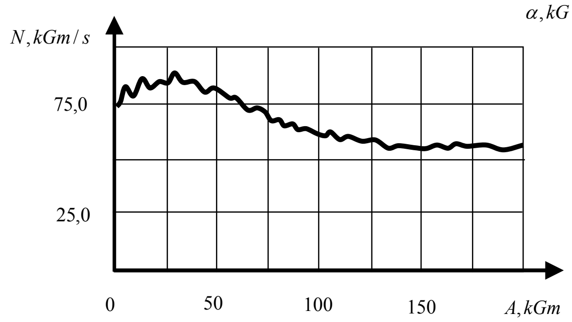
$$\hat{N} = \frac{1}{k} \sum_{i=0}^{i=k} N(i) \,, \tag{16}$$

where $\hat{N} = const.$ Than functional (15) can be simplified

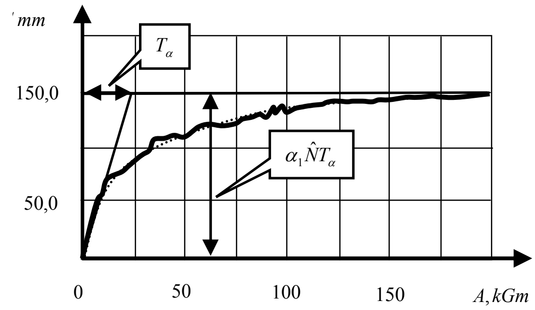
$$I = \sum_{i=0}^{i=k} \{\alpha_{\Sigma}(A_i) - \alpha_1 \hat{N} T_{\alpha} (1 - e^{-\frac{A_i}{T_{\alpha}}})\}^2 = \min \,. \tag{17}$$

Let's study the example of estimating parameters of integral operator when aperture is chiseled out in magnesium-aluminum-zinc alloy ML – 5, diameter is $d = 65{,}4mm$ at settled cutting modes. Cutting speed $V_R = 60{,}0 m/\min$, value of supply per turn $S_R = 0{,}01 mm/r$, cutting depth $t_R = 2{,}0mm$. The instrument T15K6 is made of three-side non-resharpened plates. Measurement of elastic deformations of chiseled out arbors in radial direction was made with direct strain-gauging of their curved deformations. There is a sample of characteristic of function of changing the power while working on fig. 6 «a», and function of changing $\alpha_{\Sigma}(A)$ - on fig. 6 «b». It's rather easy to estimate on following diagrams the parameters of kernel of integral operator. They are equal to $T_{\alpha} = 24kGm$, $\alpha_1 = 1{,}08 s/kGm^3$. In that case the value of the evolutional component of dynamic rigidity of cutting process is measured in kG/m. While implementing the diagnostic and management systems the usage of mathematical models for stocking the evolutional transformations is not sufficient. This is just imitation model, which allows to estimate the general trend of executive motion trajectories changes. It's reasonable to consider also the experimentally estimated trajectories of evolution of characteristical polynomial roots.

As an example of constructing the trajectories for the case of chiseling out the apertures in magnesium-aluminum-zinc alloy ML – 5 with diameter $d = 65{,}4mm$ (fig. 7). The work of cutting forces varied from zero to $300kGm$. There are only meaningful roots in the frequency range of $100Hz - 2{,}0kHz$ on the diagrams. These roots form the N^6 space, in which the separate stages of the system evolution are displayed. As it's impossible to graphically illustrate the 6D space, there are also the spatial diagrams of changing of root modules $p_i = a_i \pm b_i$ ($i = 1{,}2{,}3$), i.e. $|p_i| = \sqrt{(a_i)^2 + (b_i)^2}$ and arguments, which in this case characterize the frequencies of the major oscillators, included into AP model of a signal of fibro-acoustic emission.



«a»



«b»

Fig.7. The sample of simplified method of estimation of parameters of kernels of integral operator: «a» phase trajectory of work-power; «b» - function of transformation of summarized rigidity on work

Earlier it was shown, that the transformations of rigidity and dissipation matrixes cause the variations of both modules and arguments. Though, modules, characterizing coefficients of fading of each oscillator mostly characterize the current dissipative influence of the cutting process for vibrations, but displacements of the frequencies (root arguments) mainly depend on variations of dynamic rigidity of the cutting process.

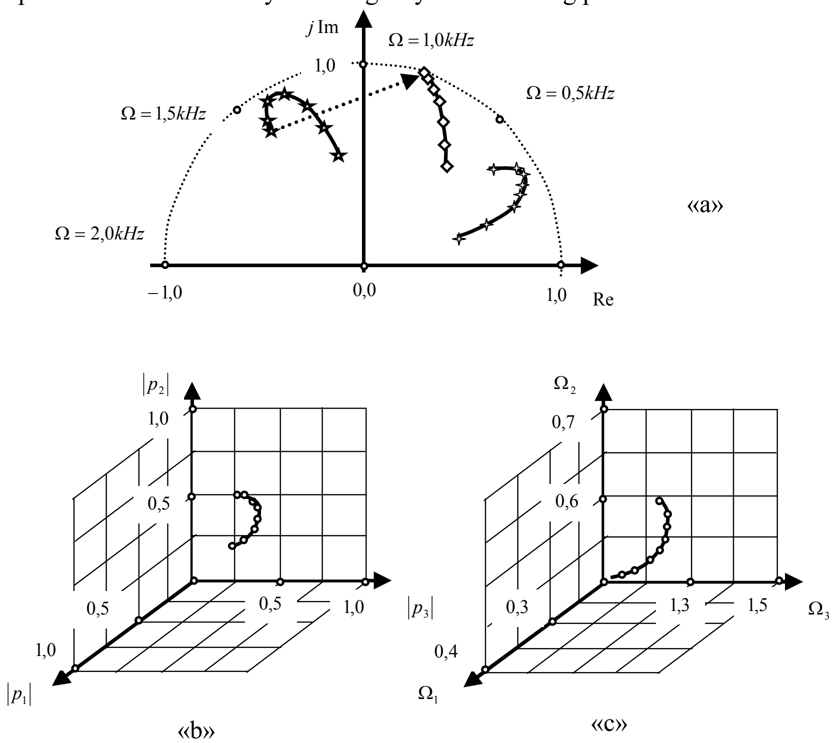


Fig. 7. The sample of evolutional diagrams of the roots of characteristical polynomial: «a» - diagram of evolution of the roots of characteristical polynomial AP of the model in complex surface; «b» - Evolutional diagram of the root modules; «c» - evolutional frequency diagram

The above mentioned diagrams are typical for cutting processing. The feature of the trajectories is that the second root reaches the single circle and the third root is being drawn up to the second one at the final stage (shown on fig. by dotted line). This kind of root transformation is caused by bifurcations of the dynamic cutting system after crossing the single circle by the second root. In this case the amplitude of the second oscillator increases times and the vibrations of the third one are being suppressed. Here let’s note, that the evolutional diagrams, which had been estimated experimentally, do characterize the data storage, which can be used for practical estimation of separate stages of the evolution. Such estimation is not complex, as it’s based on simple to be measured timely vibration consequences.

Conclusion

The above data let make the following conclusions.

1. While functioning of the cutting system its' dynamic reconstruction takes place, which is caused by evolutional transformations of parameters of dynamic characteristic of the processing. The evolutional trajectories of parameters of dynamical characteristic correspond with the trajectories of the roots of characteristically polynomial in the complex surface. Studying the trajectories of the roots within the cutting process is suitable to be worked out on the basis of autoregressive spectral analysis of the fibro-acoustic emission signal. The root trajectories characterize in their turn the information stock, which had not been used earlier, what lets diagnose and manage the cutting process online.

2. The evolution of parameters of dynamic characteristic of the cutting process causes changing of parameters of varieties, which are formed around the stationary trajectories, which are settled by trajectories of "slow" motions of executive machine elements. In some points of evolutional trajectories the changes of topology of phase space of "fast" motion subsystem are noticed, i.e. bifurcations, which change the dynamic mode of the cutting process principally. Finally, the displacing of the trajectories of form generating motions of the instrument towards the detail are noticed. They define the values of geometrical quality of the details. That is why studying of the trajectories of the roots of characteristical polynomial allows to estimate the current values of geometrical quality of the details directly while processing.

3. While modeling of the evolutional transformations in dynamic cutting system it's necessary to use the functional equations, containing parameters of dynamic characteristic related with the trajectories of space work – power with integral transformation. As the result, on one hand the forces of irreversible transformations while cutting are dependable on trajectories, on the other hand – the trajectories themselves are dependable on forces as they are defined by trajectories of work and power.

4. The materials are the basis for synergetic management of the cutting process, which is based on natural reconstructable dynamic system properties. Trajectories of executive elements of the machine characterize in this case parameters of managing the system properties with variations relative to stationary trajectories of "slow" motions, which it's necessary to choose correctly to enable the natural system properties ensure required form-generating motions of the instrument towards the detail. Integral operators, used in the dynamic equations, allow to choose between these trajectories the ones, which meet the criteria of minimum of energy losses per processing.

References

[1] A.M. Dalskii Technological providing of reliability of high-precision parts of machines, Moscow: Mashinostroenie, 1975. Page 222.

[2] Technological foundations of providing quality of machines // edited by academician of USSR Academy of Science K.S. Kolesnikov / Moscow: Mashinostroenie, 1990. Page 256.
[3] N.A. Bushe. Friction, wearing and fatigue in machines. Moscow: Transport, 1987. Page 223.
[4] A.A. Kolesnikov. Synergetics theory of management. Moscow: Energoatomizdat, 1994. Page 344.
[5] V.L. Zakovorotnyi, V.P. Bloxin, M.I. Alekseichik. Introduction in dynamic of tribosystems. Rostov-on-Don: InfoServis, 2004. Page 680.
[6] V.L. Zakovorotnyi, Attractors of mechanical systems, interacted with medium. Izvestiya TRTU. Subject issue. Synergetics and problems of control. Taganrog. TRTU publishing house, 2001. №5 (23), pages 132 – 152.
[7] V.L. Zakovorotnyi, D.A. Voloshin, A.D. Lukyanov, M.B. Flek. Modelling of wearing process of tool with the help of integrated operators. Part 1. STIN, Moscow, "STIN" publishing house, №3, 2004, pages 9-14. Part 2. STIN, Moscow, "STIN" publishing house, № 4, 2004, стр. 18-21.
[8] V.L. Zakovorotnyi, E.V. Bordachev, M.I. Alekseichik. Dynamical monitoring of conditions of a cutting process. STIN. – 1998, № 12. pages 7-14.
[9] V.L. Zakovorotnyi, I.V. Ladnik. Construction of an informational model of dynamic system of cutting machine for diagnose of machining process. Problems of engineering industry and reliability of machines. – 1991. -№ 4.
[10] V.L. Zakovorotnyi, E.V. Bordachev. Informational providing of a system of dynamic diagnose of wearing of cutting tool on the example of turning. Problems of engineering industry and reliability of machines. 1995. - №3.